

Staking, Token Pricing, and Crypto Carry*

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Abstract

The phenomenal rise of cryptocurrencies and decentralized finance have prominently featured “staking”: Besides offering a convenience yield for transactions as digital media of exchange, tokens are frequently staked (and slashed) for base-layer consensus generation or for incentivizing economic activities and platform development, and consequently earn rewards akin to deposit interests. To provide insights into the economics of staking and its asset pricing implications, we build a continuous-time model of a token-based economy agents heterogeneous in wealth stake tokens to earn rewards or use tokens for transactional convenience, all while dynamically solving their wealth allocation and consumption problem. We cast the model as a mean field game with individual stochastic controls and highlight aggregate staking ratio as a key variable linking staking to token pricing and equilibrium reward rate. The model uses transaction convenience to rationalize violations of the uncovered interest rate parity and significant carry premia in the data (e.g., a long-short carry yields a Sharpe ratio of 1.6). We relate cryptocurrencies to other major asset classes such as currencies and commodities and empirically corroborate model implications. In particular, staking ratios capture liquidity and market depth, and its correlation with reward rates is positive in the cross section but negative in the time series. Higher reward rates attract greater future staking, increasing individual’s staking allocation and the staking ratio in aggregate, which in turn predicts positive excess returns.

Keywords: Blockchain, Block Rewards, DeFi, PoS, Yield Farming, Tokenomics

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1 Introduction

The past decade witnessed the explosive growth in cryptocurrencies, which totaled 2.5 trillion USD by the end of 2021) and rising excitement about Decentralized Finance (Harvey et al., 2021), which totaled over 130 billion USD as of Feb 2022. The world is just starting to understand the categorization of tokens and their valuations as digital assets (e.g., Cong et al., 2021d; Cong and Xiao, 2021). The recent phenomenon of token staking both for base layer consensus formation and value locking for higher layer DeFi innovations further calls for a unified framework to understand the use of tokens as transaction media, investment assets, and deposit-like instruments for earning rewards.

To this end, we build a continuous-time model of an economy with token-based digital networks, where agents heterogeneous in wealth optimally conduct transactions on a (blockchain) platform, stake tokens to earn rewards, and consume off-chain. Tokens derive value by enabling users to conduct economic transactions on the digital platform, making them a hybrid of money and investable assets. Stakable tokens further serve as collateral and claims to cash flows. Our model captures the following two distinguishing features of Proof-of-Stake consensus protocols and stakable projects. First, such tokens are used on platforms that support specific economic transactions or broader use of on-chain based projects. This generates transaction convenience for holding tradable tokens as Cong et al. (2021d) proposed. Second, the rate of staking rewards that an agent earns is influenced by other agents' behavior in aggregate, but individuals take it as given when making decisions.¹

The equilibrium reward rate then is a fixed point determined by the whole distribution of heterogeneous agents each solving an optimal dynamic control problem taking the market states as given. We apply the mean field game solution to solve for the equilibrium, which marks a novel application of the methodology in economics beyond macroeconomics and inequality issues. We show that staking ratio, defined as the ratio of aggregate tokens staked in the economy to the total amount of tokens in supply, proves to be crucial for token pricing and reward rate determination in equilibrium. Related to the TVL (total value locked)

¹Take Polkadot (DOT) constitutes an example, the current reward rate for validators is determined by the current aggregate staking ratio. The less DOT is being staked, the higher are the yield for the planned amount of rewards.

ratio that practitioners emphasize, it constitutes a new predictor for token price dynamics. Uncovered interest rate parity (UIP) is naturally violated and our model predicts profitable trading strategies utilizing the concept of carry. We then empirically corroborate model predictions in a comprehensive data covered all major stakable tokens and DeFi projects.

Specifically, agents in our model derive utilities from consumption over an infinite horizon with time discount. They allocate and adjust their holdings of staked tokens, tradable tokens, and numeraire under budget constraint, trading off staking rewards, transaction convenience, and numeraire convenience for off-chain consumption. Transaction convenience endogenously increases in platform productivity, which stochastically evolves, while staking reward rate is jointly determined by aggregate reward and tokens staked. The staking ratio involves the weighted average of individual staking choices, which in turn is shaped by the agents' wealth distribution. The resulting reward rate also naturally affects agents' wealth dynamics by altering their opportunity sets. Therefore, agents' individual dynamic optimizations interact and co-evolve with the wealth distribution in a mean field game.

The resulting equilibrium is characterized by a system of partial differential equations describing the dynamics of the agent's value function (HJB equations) and the transport of wealth distribution (Fokker-Planck equation), which are connected through the fixed point problem in reward rate. Token price dynamics are fully endogenous and is described by a partial differential equation akin to the Black-Merton-Scholes formula. We simplify the equation to an ordinary differential equation concerning the total token valuation, subject to intuitive boundary conditions such that tokens are worthless for unproductive platforms and are worth the entire wealth in the economy if the platform is infinitely productive.

We derive three main model implications concerning the economics of staking and its asset pricing implications. First, staking reward is positively related to the staking ratio in the cross section. More tokens used as rewards result in a higher staking ratio in aggregate and for individual agents. Second, the expected price appreciation increases with the aggregate staking ratio, and both the price drift and staking ratio are functions of platform productivity. Third, there are general predictable excess returns to staking over holding the numeraire, which arise as a compensation for the losses in transaction and consumption convenience.

The third implication provides a perspective that links stakable tokens and compares them to traditional assets such as currencies. The token price can be treated as exchange rate against numeraire (such as USD), whereas the reward rate can be viewed as a deposit interest rate of token. Then the model implies that the classical uncovered interest rate parity (UIP) fails.² We also derive the expression for crypto carry following the general definition of carry (Koijen et al., 2018). Higher carry (equivalent to higher reward rate) attracts greater staking, which generates excess price appreciation. As an aggregation of reward rate and price appreciation, the excess return is therefore higher. Considering the reward distribution mechanism of staking economy, reward rate is automatically reduced by an excessive staking ratio. As a result of this effect, carry predicts less excess returns in the time series than in the cross-section.

For our empirical analyses, we obtain data on 60 tokens from stakingrewards.com that cover all major stakable cryptocurrencies from July 2018 to February 2022. Our empirical findings support the model predictions. First, we document that higher reward for staking significantly relate to higher staking ratio. As the aggregate reward (relative to the total amount of tokens) increases by 0.1 unit, the corresponding staking ratio increases by 8.0%. Moreover, reward rate has predictable effect on changes in staking ratio in both cross-section and time series. On average, 1% increase in reward rate in the previous week increases the staking ratio in the following week by about 0.033%. This property is robust with both two-way fixed effect and control variables including market value effect and volatility effect. However, the significance of this effect decreases with longer time intervals, reflecting to some extent the mechanical downward adjustment of the reward rate when more tokens are staked and the fact that in practice, tokens are locked for a prolonged period of time. More staking and token locked now mechanically decreases the reward rate because the staking rewards have to be divided among more staked tokens.

We also verify in the data that a larger staking ratio predicts greater price appreciation in subsequent weeks. If the staking ratio increases by 1%, the corresponding token price will

²UIP implies that the expected returns on default-free deposits across currencies are equalized, and thus the expected excess return should be zero. However, there are predictable excess returns that arise as a compensation for convenience loss. This explanation shares similar idea with Valchev (2020).

appreciate by 0.221% in the following week. Considering that the variation of staking ratio is often large, especially in the cross section, this effect is relevant for investment decisions. Crypto market return and size factors do not explain the predictive power of staking ratio, which is closely related to market liquidity and depth, and reflects the fact that tokens can be commodity-like.³ Staking reduces the supply of liquid cryptocurrencies, and hence pushes up the token's prices and increases the convenience yields of tokens. This is similar to how under capital constraints, using commodities as collateral for raising funds increases the spot price and the convenience yield of the underlying commodities (Tang and Zhu, 2016).

Finally, to test for violations of UIP, we follow Fama (1984)'s method and obtain the estimated β is significantly deviates from zero, and even close to -1 , where β should be zero under the UIP. As a direct corollary, we construct a carry trade strategy that goes long high-carry cryptos and shorts low-carry assets. The carry trade yields a Sharpe ratio of 1.6, which proves that in the cross-section, assets with higher carry generate greater returns. We further test how crypto carry predicts excess return. The result shows that high carry predicts excess returns almost one-for-one in cross-section, but the effect is reduced in time series. Intuitively, a higher reward rate attracts more staking which persists over time because tokens are typically locked for an extended period of time, reducing the reward rate going forward and thus the total expected return.

Our study adds to the literature on blockchain economics and cryptocurrency markets.⁴ In particular, we build on the tokenomics framework of Cong et al. (2021d) and Cong et al. (2021c) to add to emerging studies on Proof-of-Stake protocols (e.g., Fanti et al., 2019; Saleh, 2020; Benhaim et al., 2021) and debates on the environmental and scalability issues associated with Proof-of-Work (PoW) protocols (e.g., Cong et al., 2021e; Hinzen et al., 2019). The most closely related paper to ours is John et al. (2021) who examine equilibrium staking

³Commodities Futures Trading Commission (CFTC) regards cryptocurrencies as commodities, see, e.g., https://www.cftc.gov/sites/default/files/2019-12/oceo_bitcoinbasics0218.pdf.

⁴Extant studies mostly examine issues related to consensus algorithms (Biais et al., 2019; Saleh, 2021), cryptocurrency mining (e.g., Cong et al., 2021e; Lehar and Parlour, 2020), scalability (e.g., Abadi and Brunnermeier, 2018; John et al., 2020), fee designs Easley et al. (2019); Basu et al. (2019); Huberman et al. (2021), DeFi (e.g., Harvey et al., 2021; Capponi and Jia, 2021), ICOs (e.g., Lyandres et al., 2019; Howell et al., 2020), pricing of crypto assets (e.g., Liu et al., 2019; Cong et al., 2021a), manipulation and regulation (e.g., Griffin and Shams, 2020; Li et al., 2021; Cong et al., 2021b, n.d.), or digital currencies (e.g., Gans et al., 2015; Bech and Garratt, 2017; Chiu et al., 2019; Cong and Mayer, 2021).

on PoS blockchains and conclude that staking levels and block rewards could exhibit non-monotone relationship when agents having different trading horizons. We differ in that we focus on agent heterogeneity in wealth instead, and on equilibrium staking ratio. We also examine UIP violations and crypto carry concerning the cross section of tokens and provide empirical evidence to corroborate our model predictions. Our study is among the first to analyze equilibrium DeFi staking, complementing overviews such as Harvey et al. (2021).

A sizable literature document violations to uncovered interest rate parity (e.g. Fama, 1984; Lustig et al., 2019). Carry and its predictability have been analyzed not only for currencies but also for other assets such as equities (e.g., Fama and French, 1998; Griffin et al., 2003; Hou et al., 2011), bonds (e.g., Ilmanen, 1995; Barr and Priestley, 2004), and commodities (e.g., Bailey and Chan, 1993; Casassus and Collin-Dufresne, 2005; Tang and Xiong, 2012). Koijen et al. (2018) applies a general concept of carry and find that carry predicts returns in both the cross-section and time series. We add by documenting UIP violation and carry among cryptocurrencies (and fiat currencies). We theoretically rationalize the observations and link carry to tokenomics, complementing recent empirical work by Franz and Valentin (2020) documenting deviations of covered interest parity in cryptocurrencies.

The remainder of this paper is structured as follows. Section 2 describes the institutional background and stylized facts concerning staking. Section 3 proposes a dynamic model of the staking economy. Section 4 solves the model and derives several implications. Section 5 presents corroborating empirical evidence. Section 6 further discusses crypto carry. Section 7 concludes.

2 Institutional Background, Data, and Stylized Facts

We describe institutional background, data, and stylized facts about the staking, which involves two broad categories of activities, namely those related to pan-PoS consensus protocols and those in higher layer DeFi applications.

2.1 Staking Mechanisms

Consensus generation in PoS. Fundamentally, blockchain functions to generate relatively decentralized consensus to enable economic interactions such as value or information exchanges (e.g., Cong and He, 2019). Permissionless blockchains with Bitcoin as the best known example have historically relied on variants of the PoW protocol. Because of scalability and environmental issues of PoW (Cong et al., 2021e; John et al., 2020), PoS protocols have gained popularity and momentum for both permissioned and permissionless blockchains, with major market players adopting and incumbents such as Ethereum contemplating a conversion (Irresberger et al., 2021).

Under PoS, agents who stake native tokens have opportunities to append blocks and earn block rewards and fees as compensation. There are mainly two ways to participate. The first is to *run a validator node, staking pool, or masternode* by holding native tokens and incur the costs including hardware costs and time spent on maintenance. The more one stakes, the more likely one is selected and compensated for (Saleh, 2020, contains more details). Note that holding a token does not necessarily mean participating in staking. The second way is through *delegation*. Agents only need to delegate their tokens to an existing node or a pool and receive a reward earned by the node/pool. This route is flexible and friendly for players with less tokens and allows them risk sharing (Cong et al., 2021e). In practice, agents incur negligible physical costs (as opposed to the high entry cost in PoW mining or maintaining a node in PoS directly). In our study, we include all protocols using pan-PoS protocols such as Proof-of-Credit (POC) used in Nuls, which are variants of the above mechanisms.

Staking (value lockup) in DeFi. Incentivizing desirable behavior and guarding against misbehavior are crucial in DeFi applications. To this end, staking programs are popular and important in practice, which applies to a balance of tokens under custody in a smart contract. Users on DeFi platforms receive staking rewards as a form of interest payment from their token balance staked (Harvey et al., 2021). Synthetix is an example of an open-source DeFi protocol on Ethereum involving staking in its SNX tokens. Users can create and trade derivative tokens and gain exposure to assets like gold, bitcoin, and euros without

having to actually own them. These derivative assets are collateralized by the platform tokens (SNX) which, when locked in the contracts, enables their issuance. In return, each transaction generates a small fee distributed to SNX collateral providers. Another example is ChainLink, the leading decentralized oracle network. Oracle nodes stake LINK tokens to compete for service tasks and to ensure truthful reporting while depositors stake tokens to help with the alert system for bribery resistance and network security. In return, these agents earn staking rewards from both newly issued tokens and fees.

Without getting bogged down with specific threshold requirements and operational differences across various DeFi protocols and smart contracts, DeFi staking can be characterized as simply having different benefits (rewards) and costs (including risks), from the stakers' perspective. Overall, staking shares the spirit of certificates of deposit or risky illiquid investments.

Reward determination and slashing. In most staking programs, including PoS chains, on-chain projects and DeFi platforms, the total amount of rewards used to incentivize staking or its determination mechanism is pre-specified and announced. Therefore, the aggregate reward for a specific window of time is common knowledge.

In PoS, blockchain branch is randomly selected from the whole staking pool. That is, staking reward is randomly distributed to stakeholders based on the number of staked coins they hold as a probability weight. For example, if investor stake 10 coins while the aggregate staked amount of this branch is 100, then the investor has a 10% probability of appending to the branch and receiving staking reward. As the above process is continuously repeated, we can calculate the expected reward by multiplying the aggregate reward and the probability. Similarly, in DeFi platforms, stakers share the rewards that come from transaction fees or pre-determined emissions.

Since the rewards are yield from staked tokens, *staking reward rate* is naturally compared to interest rate. However, unlike deposit rates set by the banks, staking reward rate is jointly determined by announced staking reward and the aggregate tokens staked. Appendix E. details the staking programs for the tokens in our sample.

In addition to the opportunity costs, stakers also risk losing the staked tokens due to possible security attack, illegal verification, and storage failure. In order to discourage validator misbehavior, most projects propose a punishment mechanism known as *slashing*. A pre-defined percentage of a validator's tokens are lost when it does not behave consistently or as expected on the network. The two prominent cases causing slashing are downtime and double signing, with the latter involving much larger penalties typically.

Market and information. In PoS, validators compete in the staked amount to earn reward. To incentive more delegates, they develop a reward distribution plan at the node level. Potentially delegators can freely choose among these nodes or delegate through some intermediaries. Therefore, nodes engage in price competition for delegated stakes.⁵ For DeFi platforms, staking reward rate are typically equal for participants, but some white-listed groups may have priority in staking. Most stakable tokens are launched on mainstream cryptocurrency exchanges. Investors can easily invest in these staking projects and trade these tokens with cryptocurrency assets such as Bitcoin and Ethereum.

Information on staking programs, including participation rules, reward distribution plan, total staked value (or total value locked, TVL, which includes non-native tokens), and even information of all the validators, are open and can be easily obtained on official websites of projects. Third party websites also specialize in collecting real-time information on staking projects; examples include *Stakingrewards.com* and *EarnCryptoInterest*. In particular, an important variable in our analysis, the *staking ratio*, which captures the total number of tokens staked as a fraction of the total number of tokens, is public knowledge.

2.2 Data

Our acquire data from *Stakingrewards.com*, one of the largest websites that collect information on staking and offer both historical and real-time data of most stakable assets. Stakable asset is a general classification for tokens with properties to staking, which includes those on PoS-type blockchains (e.g., NPOS and POC) and on-chain projects which enable

⁵In practice, staking reward rates may not exactly equal. For example, large nodes may have slightly lower reward rates because they are considered more reliable and have more stable returns.

passive returns through staking. The information about staking reward and proportion is typically aggregated from official websites of each token, including staking participation methods, reward sharing rules, real-time staking amount (staking ratio), yield calculation, etc. Price data come from mainstream crypto markets. Note that there may exist multiple staking participation methods for one token. We always choose the participation method with the lowest capital threshold and risk, such as delegating, voting, etc. Please see Appendix E. for details about staking participation.

Our sample covers daily observations of 60 stakable tokens with the largest market capitalization and longest time span. The sample period covers July 2018 through Feb 2022, which overlaps with the initial birth and rapid growth of “staking.”⁶ Our sample covered not only the top stakable assets with the greatest market value as of the latest date (Feb 2022), but also all the stakable assets with a market value of more than 100 million US dollars as of the earlier period (Aug 2020), including POS chains (e.g., Solana), pan-POS protocol (e.g., Nuls), on-chain projects (e.g., Matic), and DeFi applications (e.g., SNX).

Table 1 contains summary statistics of the tokens. In most of our analysis, we aggregate the daily observations into weekly data because daily data contain much more noise. We also aggregate data into 14-day and 30-day windows for robustness. The summary statistics show a large dispersion in the status of staking participation and price appreciation among tokens: the mean staking reward rate ranges from 0.02% to 75.20%, while the mean staking ratio ranges from 6.30% to 98.02%.

2.3 Empirical Patterns in Staking and Token Pricing

Aggregate trends. The staking economy has grown rapidly in recent years. First, for layer 1, the shift of focus away from PoW and onto the PoS consensus algorithms has been evident and timely.⁷ The PoS share, has increased substantially over time from 5% in October of 2019 to over 20% in October 2021. As of Oct. 2021, the PoS market cap is \$

⁶Some tokens were launched after the start date, thus the sample set is an unbalanced panel.

⁷According to 2021 Staking Ecosystem Report 2021 published by *StakingRewards* in Oct. 2021, available at <https://cms.stakingrewards.com/wp-content/uploads/2021/10/2021-Staking-Ecosystem-Report-1.pdf>.

326.775 Billion, up from \$ 21.117 Billion a year ago. The annual growth rate reached 1,500%, while the overall crypto market cap is up by 673%.

Meanwhile, there are more than 60 stakable DeFi assets, 27 masternodes and more than 50 mainstream crypto assets that can be staked for rewards on DeFi platforms by the end of 2021. The entire staking economy has grown to over 4 million total users. Stakers earn a weighted-average 8% (or an equal-weighted 15%) annual staking reward rate approximately with a 40.91% weighted-average staking ratio.

Violations in uncovered interest rate parity. The Uncovered Interest Rate Parity (UIP) is an important benchmark in traditional international exchange models, especially in exchange rate determination. It implies that the difference in interest rates between two countries will equal the relative change in currency foreign exchange rates over the same period. However, UIP violation is widely documented in empirical studies (e.g., Backus et al., 1993; Engel, 1996, 2016): An increase in the domestic interest rate relative to the foreign one is associated with an increase in the excess return on the domestic currency over the foreign currency (the “UIP Puzzle”). Many explanations for the UIP violation have been proposed in previous studies, ranging from time-varying risk including liquidity and volatility risk (e.g., Bekaert, 1996; Verdelhan, 2010; Gabaix and Maggiori, 2015; Lustig et al., 2011), peso problems (e.g., Burnside et al., 2011), to time-varying convenience yield differentials (e.g., Valchev, 2020; Jiang et al., 2021).

Table 1: Summary statistics: staking reward rate, staking ratio and price change.

Token	Reward Rate, r (%, Annual)		Staking Ratio, Θ (%)		Daily Return (%)		Token	Reward Rate, r (%, Annual)		Staking Ratio, Θ (%)		Daily Return (%)	
	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.		Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.
linch	3.23	5.99	13.49	9.24	-0.39	6.32	kyber	4.27	2.44	27.08	2.98	0.18	5.65
aave	4.16	0.97	23.21	1.80	0.30	6.35	livepeer	63.63	31.17	63.64	4.70	-0.27	11.48
aion	6.57	1.32	26.10	3.28	0.17	6.72	lto	7.31	1.23	24.59	4.40	0.67	7.14
algorand	6.54	3.10	58.01	9.15	0.03	6.03	matic	24.13	12.50	24.96	3.81	0.76	7.38
ark	9.29	0.52	54.55	1.40	-0.04	6.27	mina	12.11	1.24	98.02	4.48	-0.18	7.38
avalanche	10.11	2.17	65.78	8.49	0.65	7.39	mirror	42.09	24.44	32.88	7.21	-0.32	6.10
band	12.99	1.54	80.36	1.64	0.20	6.79	near	11.98	2.01	36.77	4.40	0.45	7.14
bifi	8.38	3.33	47.47	15.67	0.21	6.83	nem	0.02	0.01	41.39	1.41	-0.16	4.78
binance-sc	12.52	5.19	67.99	12.62	0.16	4.54	neo	2.53	1.29			0.05	5.45
bitbay	2.25	0.50	45.86	6.68	0.96	19.71	nuls	9.17	0.97	44.26	4.05	0.17	6.89
cardano	5.96	2.13	66.10	10.24	0.35	5.83	oasis	15.72	2.24	48.22	3.43	0.12	6.97
cosmos	9.68	1.32	67.93	6.52	0.15	6.22	pancakeswap	75.20	26.27	37.89	2.57	0.07	6.85
curve	3.22	1.79	73.78	19.06	0.08	7.01	peakdefi	44.14	16.63	34.66	24.78	-0.29	5.75
dash	6.28	0.28	52.45	2.45	-0.09	5.62	polkadot	12.18	1.76	60.59	5.38	0.42	6.18
decred	6.48	1.84	54.26	4.57	0.13	5.14	qtum	5.68	0.87	16.86	2.49	0.09	5.26
dfinity	15.17	2.14	49.47	0.50	-0.74	5.71	secret	26.95	2.62	49.48	4.34	0.50	7.49
dodo	67.60	4.56	42.47	3.85	-0.88	5.83	smartcash	2.49	0.53	7.77	0.77	0.12	6.40
elrond	17.91	7.00	53.42	6.88	0.64	6.33	snx	26.56	21.19	62.41	15.12	0.33	6.37
eos	4.67	3.72	38.08	23.83	0.02	5.06	solana	8.41	2.86	70.87	8.57	0.67	6.68
eth2.0	11.61	11.87	6.30	2.62	0.46	3.89	stafi	20.94	3.20	21.17	3.33	-0.04	8.12
fantom	30.30	25.39	58.27	9.77	0.66	8.79	stake-dao	22.74	9.09	34.70	4.83	-0.14	8.16
flow	9.31	1.13	57.87	10.58	-0.63	5.24	sushi	10.44	4.57	31.78	3.00	-0.35	6.44
harmony	10.54	0.77	45.29	6.03	0.59	7.38	terra	9.35	3.36	31.29	4.66	0.63	6.53
icon	17.42	3.46	27.30	6.52	0.20	6.15	tezos	6.58	1.39	72.36	9.21	0.05	5.76
idex	6.89	6.81	34.09	12.78	0.27	7.54	tron	3.79	1.28	24.64	5.00	0.14	5.08
injective	4.35	0.18	97.34	0.83	-0.38	6.55	wanchain	8.12	0.56	24.77	1.66	0.19	6.24
iotex	10.15	3.62	41.81	6.17	0.09	6.62	waves	4.39	1.44	64.61	11.87	0.24	5.99
irisnet	10.64	0.59	33.83	1.92	0.40	8.12	wax	3.00	1.95	61.08	22.38	0.29	6.05
kava	18.66	19.30	66.36	10.89	0.32	6.30	yearn	4.75	2.68	8.35	9.26	0.32	6.97
kusama	14.14	0.79	53.58	10.81	0.52	6.38	zcoin	16.04	3.28	56.88	10.18	0.35	4.72

Notes: According to the reward distribution mechanism, there is no concept of staking ratio for *neo*.

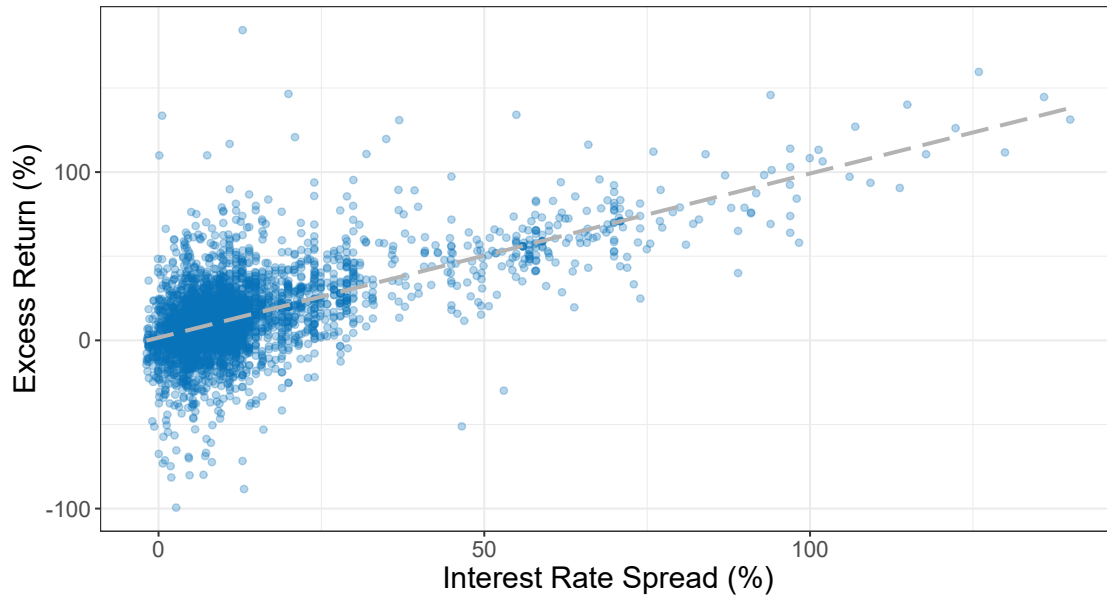


Figure 1: UIP violation in the staking cryptocurrency market.

This figure empirically visualizes the violation of uncovered interest rate parity (UIP) in the cryptocurrency market based on the data as Table 1 summarizes. We treat the US dollar as local currency and the 1Y treasury interest rate as the local interest rate. The data is from the Federal Reserve. For the tokens in our sample, we convert the data into weekly data. Each point in the figure indicates a weekly data point for a particular token. The staking reward rate (annualized) is used as foreign interest rate, and the x-axis, the interest rate spread, is calculated as the foreign interest rate minus the local interest rate. The y-axis is the excess return in the next week, which includes the interest rate spread and the price appreciation. Since the token are priced in US dollars, the price change is the change in foreign exchange rates. The grey line shows a linear trend of the scatter points. If UIP holds, the slope should be close to zero. However, the points and the grey line are significantly sloping upwards, implying that an increase in the foreign interest rate relative to the local one is associated with an increase in the excess return on the cryptocurrency over the local currency, i.e. which is so-called “the UIP puzzle”.

We find that UIP is also violated in cryptocurrencies. Since token price and staking reward rate can be compared to exchange rates and interest rates, we can directly document crypto UIP violations. Specifically, if we treat the U.S. dollar as a local currency, then the change of token price (denominated in US dollar) is equivalently considered as the change in foreign exchange rates. Moreover, earning staking reward is similar to earning interest rates. Figure 1 illustrates with a plot of the excess return in the next week against the interest rate spread calculated as the “foreign interest rate” minus the “local interest rate.” Each blue circle in the figure indicates a weekly data point for a particular token, and the grey line shows a fitted line. If UIP holds, the slope should be close to zero. However, the observed upward slope implies that an increase in the foreign interest rate relative to the local one is associated with an increase in the excess return on the cryptocurrency over

the local currency, i.e., the so-called “the UIP puzzle.” We discuss the implications of UIP violations further after we introduce our model of the staking economy.

3 A Dynamic Model of the Staking Economy

In our model, heterogeneous agents optimally allocate individual wealth in a continuous-time economy with a digital network subject to productivity shocks. The native token adoption and staking reward rate are endogenously determined. We capture the interaction among agents’ stochastic control and the evolution of aggregate system states using a mean-field game-theoretical approach, which has been used in describing, e.g., trade crowding (Cardaliaguet and Lehalle, 2018) and mining competition (Li et al., 2019). We also introduce an important state variable, the staking ratio, defined as the ratio of tokens staked in the economy to the total amount of tokens. It is an aggregation of the agents’ controls as well as an important system state that influencing token prices and agents’ optimizations.

3.1 Setup

Time is continuous and infinite. A continuum of agents conducts peer-to-peer transactions on a blockchain platform or a general digital marketplace while participating in staking programs either to provide network consensus or contribute to certain DeFi protocols. A generic consumption good serves as the numeraire and medium of exchange on the platform is its native token.

Platform productivity and token price. As in Cong et al. (2021d), platform productivity A_t captures the general usefulness and functionality of the digital platform, and thus reflects the convenience users obtain transacting on the platform using its tokens. We assume that A_t evolves according to a Geometric Brownian Motion:

$$dA_t = \mu^A A_t dt + \sigma^A A_t dZ_t, \quad (1)$$

where Z_t is the primary source of uncertainty for the platform economy.

Without loss of generality, we denote the token price (in numeraire) as P_t , whose dynamics follow a general diffusion process:

$$dP_t = \mu_t P_t dt + \sigma_t P_t dZ_t, \quad (2)$$

where μ_t and σ_t are endogenous and in general time-varying.

Agents, adoption, and convenience. Agents of unit measure is indexed by i and each characterized by her current wealth $w_{i,t}$. Each agent makes consumption-portfolio choices among staked (locked) tokens, non-staked (tradable) tokens, and numeraire (consumption goods or fiat). An agent becomes a platform user if she holds tokens either for staking or transactions on the platform.

Users gain convenience from holding tokens and conducting economic activities on the platform. Since staked tokens are locked from the staker's perspective, they can only derive transaction convenience from non-staked (tradable) tokens, which we model similarly as in Cong et al. (2021d,c): For an agent holding x_t (in numeraire) worth of tradable tokens on the platform, she derives a utility flow:

$$dv(x_t) = dv_t = x_t^{1-\alpha} (U_t A_t)^\alpha dt. \quad (3)$$

The marginal transaction convenience $\frac{\partial v}{\partial x} > 0$ and decreases with x_t with $\alpha \in (0, 1)$. $U_t = U(w_t) > 0$ is the user type that reflects heterogeneity in transaction needs and is a function of wealth w_t since agents only differ in w_t .

Following Bansal and Coleman (1996) and Valchev (2020), the convenience of holding the numeraire is reflected in the reduction of transaction costs in consumption. We denote the transaction cost as $\Psi_t = \Psi_t(y_t, n_t) \geq 0$, where y_t and n_t are consumption and numeraire holdings respectively. Naturally, $\frac{\partial \Psi}{\partial y} > 0$ and $\frac{\partial \Psi}{\partial n} < 0$. Then $-\frac{\partial \Psi}{\partial n} > 0$ reflects the marginal convenience yield of holding the numeraire.

We later specify that the token convenience and transaction costs enter agents' wealth dynamics rather than utility for two reasons: First, token convenience flow and transaction

costs are indeed in monetary form in practice, corresponding to business profits and liquidity costs on real balances respectively. Second, this approach is functionally equivalent to accounting them in the utility function (Feenstra, 1986), and is a standard approach in the literatures on convenience yields of bonds, etc.

Staking rewards. Staking rewards are used to incentivize agents to stake their tokens to either generate consensus records in a base layer or to participate in some DeFi program, such as a liquidity pool or insurance pool. In practice, staking rewards come from additional token issuance (emission) or fees others pay. The reward schedule is typically public information the time of staking, and can be at least estimated based on real-time blockchain data (see details in Appendix E.). To quantify staking rewards, we denote the total amount of tokens at time t as Q_t , which satisfies

$$dQ_t = E(Q_t, A_t)dt. \quad (4)$$

The growth rate of token supply implies an inflation rate ι_t . We denote the aggregate rewards generated by the transaction fee by a random variable $F_t = \tau_t Q_t \geq 0$, the randomness can capture unexpected reward shock.⁸ If the system involves only a constant emission for rewards, then the total amount of tokens distributed as staking rewards at time t , R_t , come from a combination of emission and fees:

$$R_t = E(Q_t, A_t) + F_t(Q_t, A_t) = \iota_t Q_t + \tau_t Q_t. \quad (5)$$

All staked tokens are fungible and consequently all stakers face a reward rate akin to interest rates on bank deposits:

$$r_t \equiv \frac{R_t}{L_t}, \quad \text{where } L_t \text{ is the aggregate amount of staked tokens at } t. \quad (6)$$

To capture the cost of node operation and risk of slashing, we assume that stakers incur

⁸We need no additional assumptions about F_t for both the theoretical and numerical analyses. In practice, the expected aggregate transaction fee weakly increases with A_t . In our baseline model, τ is a mean zero random variable. We also extend it to a random variable with non-negative mean $\bar{\tau}$, and even considering the case that $\bar{\tau}$ increases with A_t , which has little impact on the main properties in the present work.

costs at a rate $c_t < r_t$ proportional to their staking amount. Then if someone stakes k_t tokens ($k_t P_t$ dollars), by Itô's Lemma, the resulting wealth increments satisfy:

$$d(k_t P_t) = k_t dP_t + P_t(r_t - c_t)k_t dt = (k_t P_t)[(\mu_t + r_t - c_t)dt + \sigma dZ_t]. \quad (7)$$

3.2 Agents' Problem and Staking as Optimal Control

At time t , an agent with wealth w_t chooses the level of consumption y_t , and holds a portfolio consisting of l_t numeraire-equivalent amount of staked tokens, x_t numeraire-equivalent amount of tradable tokens and n_t numeraire, where $x_t, l_t, n_t \in [0, w_t]$, $n_t = w_t - x_t - l_t$.

Given personal wealth w_t and staking reward rate r_t , agents decide the controls (y_t, x_t, l_t) to optimize discounted infinite horizon utility.

$$\max_{\{y_s, x_s, l_s\}_{s=t}^{\infty}} E_t \left[\int_t^{\infty} e^{-\phi(s-t)} \mathcal{U}(y_s) ds \right], \quad (8)$$

where $\mathcal{U}(y_t)$ is agents instant utility from consumption, which is strictly increasing and concave, and ϕ is the discount rate. The agent's wealth dynamics has to satisfy:

$$dw_t = [(x_t + l_t)\mu_t + l_t(r_t - c_t) + v_t - y_t - \Psi_t]dt + (x_t + l_t)\sigma_t dZ_t. \quad (9)$$

Because the staked tokens cannot be traded, the agent also faces the budget constraint:

$$y_t \leq w_t - l_t. \quad (10)$$

For each given r_t , define the indirect utility function as

$$J(t, w_t; r_t) = \max_{\{y_s, x_s, l_s\}_{s=t}^{\infty}} E_t \left[\int_t^{\infty} e^{-\phi(s-t)} \mathcal{U}(y_s) ds \right]. \quad (11)$$

We can derive the Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_{\{y_t, x_t, l_t\}} \left\{ \mathcal{U}(y_t) - \phi J + f(y_t, x_t, l_t; w_t, r_t) \frac{\partial J(t, w_t; r_t)}{\partial w} + \frac{\sigma_t^2}{2} (x_t + l_t)^2 \frac{\partial^2 J(t, w_t; r_t)}{\partial w^2} \right\}, \quad (12)$$

where $f(y_t, x_t, l_t; w_t, r_t) = (x_t + l_t)\mu_t + l_t(r_t - c_t) + v_t - y_t - \Psi_t$.

3.3 Dynamic Equilibrium

We now solve for a Markovian equilibrium for the mean-field game.

Staking ratio. Denote the density function of investors' wealth w at time t as $m(t, w_t)$. Assume all the investors have non-negative and finite wealth. m is an absolutely continuous density on the state space $W = [0, \bar{w}]$. An important global variable is the staking ratio Θ_t , the ratio of the aggregate number of staked tokens to the total number of tokens under the current given system states:

$$\Theta_t = \Theta(r_t) = \frac{L_t}{Q_t} = \frac{\int_W l(t, w_t; r_t) m(t, w_t) dw_t}{\int_W [x(t, w_t; r_t) + l(t, w_t; r_t)] m(t, w_t) dw_t}, \quad (13)$$

From the agents' perspective, they make staking choices taken as given the reward rate. That is, staking ratio is a function of current reward rate r_t . Through the HJB equation and the continuity assumption of m , it can be shown that Θ_t is continuous in r_t . Staking ratio is important because it links individual choices with global states. It can be viewed and tracked on the public data websites such as *StakingRewards.com* or the official platform of the tokens. Therefore, Θ_t in public information at time t in practice.

Token market clearing. In aggregate, the total amount of tokens Q_t is equal to the sum of the number of individual's personal token holdings, i.e.

$$Q_t P_t = \int_W (x_t + l_t) m(t, w_t) dw_t = \widehat{x_t + l_t}, \quad (14)$$

where $l_t = l(t, w_t; r_t)$ and $x_t = x(t, w_t; r_t)$ are the value of staked and non-staked (tradable) token holdings respectively, and $\widehat{x_t + l_t}$ represents (wealth weighted) average value of $x_t + l_t$, which is essentially the total wealth allocated to the platform with a unit measure of agents.

Combining Eq.(13) and Eq.(14), we obtain:

$$P_t L_t = P_t Q_t \Theta_t = \int_W l_t m(t, w_t) dw_t. \quad (15)$$

This equation is only related to the staked tokens, which can be considered as the market clearing condition in the staking market. Naturally, the token price P_t that satisfies the market clearing condition should simultaneously clear both the staking and non-staking market. Otherwise, arbitrage opportunities arise.

Mean-field game equilibrium. We now consider the evolution of the mean field of this framework, i.e. the distribution of investors' wealth $m(t, w_t)$. The dynamic is characterized by Fokker-Planck equation as Eq.(16), with initial condition $m(0, w_0) = m_0$.

$$0 = \frac{\partial}{\partial t} m + \frac{\partial}{\partial w} [f(y_t, x_t, l_t; w_t, r_t) m] - \frac{1}{2} \frac{\partial^2}{\partial w^2} [(x_t + l_t)^2 \sigma^2 m], \quad (16)$$

where $f(y_t, x_t, l_t; w_t, r_t) = (x_t + l_t)\mu_t + l_t(r_t - c_t) + v_t - y_t - \Psi_t$.

The evolution of the wealth density changes the system states and, in turn, agent's optimization problem. According to the reward distribution mechanism as Eq.(6) shows, the resulting reward rate r_t is updated by the aggregate of agents' controls, $\Theta(r_t)$. In equilibrium, we obtain a fixed point problem in r_t :

$$r_t = \frac{R_t}{Q_t \Theta(r_t)}. \quad (17)$$

We denote the equilibrium reward rate and staking ratio as r_t^* and $\Theta_t^* = \Theta(r_t^*)$. We naturally define ρ_t as the staking reward ratio,

$$\rho_t \equiv \frac{R_t}{Q_t} = \iota_t + \tau_t. \quad (18)$$

It indicates the number of tokens used for rewards as a percentage of the total amount of tokens on the platform. In practice, ρ_t and r_t are both important characteristics in the staking economy. In most staking economy, especially most PoS chains, the aggregate reward

ratio is fixed or at least can be estimated, while the staking reward rate features the actual return that agents will earn like deposit rate. In contrast to staking reward rate in (6), r_t , staking reward ratio ρ_t is a system state and completely independent with agents' staking activities. Since ρ_t has a one-to-one correspondence to the equilibrium r_t^* , we write the equilibrium staking ratio as $\Theta(\rho_t)$ when performing comparative statics analysis but as $\Theta(r_t)$ in the process of deriving and characterizing the equilibrium.

Eq.(12) is a backward PDE on discounted value function J , and Eq.(16) is a forward transport equation of mean field density m with initial condition $m(0, w_0) = m_0$. Together with (17), they make up the whole mean-field game system (with controls). A mean field game (MFG) equilibrium is then characterized by agents' controls $\{y_t, x_t, l_t\}_{t=0}^\infty$ and evolutionary path of system states $\{P_t, r_t, \Theta_t, m_t\}_{t=0}^\infty$ such that each agent solve her optimization problem, the token market clearing condition is satisfied, and the system states satisfies the Fokker-Planck equation and the fixed point problem. Note that in this infinite horizon MFG system, there is no reason for a stationary state of density m as an equilibrium.⁹ The PDEs are a system of evolution equations given the initial density state. See Appendix C. for more discussion on mean field game system.

4 Model Solution and Implications

We solve the model and derive implications that hold under general wealth distributions.

4.1 Staking Ratio & Reward

We start by analyzing the optimal decision of a single agent. We define θ_t as agents' individual staking ratio given reward rate r_t at time t .

$$\theta_t = \theta(w_t; r_t) = \frac{l(t, w_t; r_t)}{x(t, w_t; r_t) + l(t, w_t; r_t)} = \frac{l(t, w_t; r_t)}{q(t, w_t; r_t)}, \quad (19)$$

where $q_t = q(t, w_t; r_t) = x_t + l_t$ is the aggregate value of individual token holding.

⁹In other words, there is no reason for $\frac{\partial m}{\partial t} = 0$ in Eq.(16) when solving the equilibrium.

At the instant of decision-making, the agent takes the reward rate r_t as given, whereas according to Eq.(3), transaction convenience is related to user type. In particular, the marginal transaction convenience is decreasing with x_t , the amount of tradable token held. Naturally, the agent makes a trade-off between obtaining staking reward and transaction convenience. Intuitively, when the reward rate is higher, the agent should have a higher individual staking ratio θ_t . Moreover, for a given user type, when the agent holds very few tokens, staking should be dominated by non-staking, since the marginal transaction convenience is sufficiently high. Proposition 1 formalizes the tradeoffs.

Proposition 1. *Optimal individual staking.* *For an agent with wealth w_t , the optimal aggregate value of token holding q_t^* is unique and positive. The optimal individual staking ratio θ_t^* is weakly increasing in w_t and satisfies*

$$\theta_t^* = \max \left\{ 0, 1 - \left(\frac{1 - \alpha}{r_t - c_t} \right)^{\frac{1}{\alpha}} \frac{A_t U_t}{q_t^*} \right\}. \quad (20)$$

Clearly, agents with different wealth will have heterogeneous optimal individual staking ratio. However, the common denominator is that when the staking reward is greater, agents' staking ratio will also be greater. (20) reflects agent's trade-off between staking reward and transaction convenience. Until the marginal transaction convenience becomes smaller than the staking reward rate, i.e. $r_t - c_t > (1 - \alpha) \left(\frac{A_t U_t}{q_t^*} \right)^\alpha$, the agent starts putting the excess token positions into the staking pool.

Substituting the agents' individual optimal choices into (13), we obtain the resulting aggregate staking ratio $\Theta(r_t)$. Intuitively, $\Theta(r_t)$ also weakly increases with reward rate r_t , but $(r_t, \Theta(r_t))$ need to be jointly determined in equilibrium. Suppose the reward rate are higher, agents will expect a larger staking ratio, which in turn leads to a decrease in r_t . $(r_t, \Theta(r_t))$ should satisfies Eq.(17). As mentioned in Section 3.3, the equilibrium is determined by the system state, the aggregate staking reward ratio ρ_t .

Proposition 2. *Equilibrium staking ratio.* *Higher total staking reward ratio leads to a higher system staking ratio in equilibrium, i.e. $\forall \rho' > \rho > 0$,*

$$\Theta(\rho') \geq \Theta(\rho), \quad (21)$$

where $\rho = \frac{R}{Q}$ is defined as the aggregate staking reward ratio (the number of tokens used for rewards as a percentage of the total amount of tokens).

Proposition 2 gives a general expression on how aggregate staking reward affects staking ratio in equilibrium. For a given platform productivity A_t , as the aggregate staking reward ratio ρ_t increases, the corresponding overall staking ratio Θ increases. Note that Proposition 2 holds for any given distribution of agents' wealth $m(t, w_t)$. the result applies for both cross-sectional comparison and time series analysis. Note that higher reward rate r_t does not necessarily lead to higher equilibrium staking ratio. Fixing the aggregate staking reward, more wealth staked implies a low reward rate.

Figure 2 visualizes the optimal staking choice of heterogeneous agents and the generation of equilibrium of the staking economy. Here we focus on comparative statics of reward rate r_t , thus the platform productivity A_t is fixed.¹⁰ To simplify the numerical solutions and focus on our main interest, we make further assumption about user type $U_t = U(w_t)$ here. Since the user type reflects the demand for transactions, we assume U_t increases monotonically and convexly with agent's wealth, $\frac{\partial U}{\partial w} > 0$ and $\frac{\partial^2 U}{\partial w^2} > 0$. This assumption is consistent with the reality that richer people have greater demand for transaction. For each agent, the individual staking ratio increases with reward rate. As for the comparison among crowds, the wealthier agents have a greater optimal staking ratio, since after they have put enough tokens into transactions, there are still wealth left to be used for staking.

As the blue curve in Figure 2 shows, the overall staking ratio $\Theta(r_t)$ weakly increases with staking reward rate r_t . Since Θ can be seen as a weighted average of θ , combined with Proposition 1, the above observation seems to be natural. In the grey interval, more agents enter the staking market as reward rate increases. When the reward rate continues to increase, all the agents have entered the market and they will gradually increase the proportion of staking. The downward sloping black curve corresponds to (17). The unique intersection (the black point in the figure) of these two curves gives the equilibrium (r_t^*, Θ_t^*) at time t . Note that the platform productivity, A_t , influences the equilibrium not only by

¹⁰In fact, as A_t and Q_t given, μ_t and σ_t is determined in our model. Here we straightly substitute the corresponding value, the detailed analysis on token pricing will be discussed in later subsections.

affecting the transaction convenience, but also through affecting the token price and its dynamics, which we discuss next.



Figure 2: Individual staking decisions and equilibrium staking ratio.

This figure shows the heterogeneous individual staking decisions. For each agent, the individual staking ratio weakly increases with reward rate. As for the comparison among crowds, the agent who owns more wealth will have a greater optimal staking ratio, since after they have invested enough tokens for transaction, there are still tokens left to be used for staking. As Proposition 1 shows, the individual staking ratio curve is a piecewise function. In the grey interval as this figure shows, more agents enter the staking market as reward rate increases. When the reward rate continues to increase, all the agents have entered the market and they will gradually increase the proportion of staking. The blue curve is the sum of the individual staking curve, which features the resulting overall staking ratio $\Theta(r_t)$. The downward sloping black curve draws the points that satisfies the equilibrium Eq.(17). As a result, the only intersection point formed by these two curves is the equilibrium situation (r_t^*, Θ_t^*) under the current system state. Here the system state A_t is fixed to be 1, $\rho_t = 3\%$. μ_t and σ_t are endogenously given by the states, which will be analyzed later. The selection of the parameters takes into account the reality of the staking market and relevant literature (details in Appendix D.).

4.2 Staking Ratio & Price Dynamics

We link staking activities to token prices. In general, the token price appreciates when more agents' wealth flows into the platform, whether it is due to high platform productivity and thus large transaction convenience, or due to greater participation in staking. We are interested in the drift term μ_t of token prices, which depends on both agents' control and

wealth distribution.

Since A_t is the only exogenous state variable, all endogenous variables are functions of A_t in equilibrium. In addition, the exogenous token supply, Q_t , also affects the price. Denote $P_t = P(A_t, Q_t)$ and apply Itô's Lemma, we obtain

$$dP_t = \left[\frac{\partial P_t}{\partial A_t} A_t \mu^A + \frac{\partial P_t}{\partial Q_t} Q_t \iota_t + \frac{1}{2} \frac{\partial^2 P_t}{\partial A_t^2} (A_t \sigma^A)^2 \right] dt + \frac{\partial P_t}{\partial A_t} A_t \sigma^A dZ_t. \quad (22)$$

By matching the coefficients to Eq.(2), we obtain

$$\begin{aligned} \mu_t &= \frac{1}{P_t} \left[\frac{\partial P_t}{\partial A_t} A_t \mu^A + \frac{\partial P_t}{\partial Q_t} Q_t \iota_t + \frac{1}{2} \frac{\partial^2 P_t}{\partial A_t^2} (A_t \sigma^A)^2 \right], \\ \sigma_t &= \frac{1}{P_t} \frac{\partial P_t}{\partial A_t} A_t \sigma^A. \end{aligned} \quad (23)$$

For each agent with a positive optimal staked value l_t^* given r_t , the marginal benefits of staking is strictly larger than marginal transaction benefits. By the F.O.C., l_t^* satisfies:

$$0 = \left(\mu_t + r_t - c_t + \frac{\partial \Psi_t}{\partial n_t} \right) \frac{\partial J}{\partial w} + (\tilde{x}_t + l_t^*) \sigma_t^2 \frac{\partial^2 J}{\partial w^2}, \quad (24)$$

where $\tilde{x}_t = \left(\frac{1-\alpha}{r_t-c_t} \right)^{\frac{1}{\alpha}} A_t U_t$. (See details in the proof of Proposition 2 in Appendix A.2.) Since the user type is only related to user's wealth, we define Σ_t a subset of the feasible domain of wealth, W . Agent with wealth w_i will obtain a positive optimal staking choice if and only if $w_i \in \Sigma_t$. By the fixed-point Eq.(17), the equilibrium staking ratio $\Theta_t > 0$, thus $\Sigma_t \neq \emptyset$. Integrating w over Σ_t and substituting into the market clearing condition Eq.(15), we obtain:

$$\begin{aligned} 0 &= \frac{1}{\sigma^2} (\mu + r - c) \int_{\Sigma} \frac{\partial_w J}{\partial_{ww} J} m dw + \frac{1}{\sigma^2} \int_{\Sigma} \frac{\partial_w J}{\partial_{ww} J} \frac{\partial \Psi}{\partial n} m dw \\ &\quad + PQ\Theta + \left(\frac{1-\alpha}{r-c} \right)^{\frac{1}{\alpha}} A \int_{\Sigma} U m dw, \end{aligned} \quad (25)$$

where the time subscript is omitted, $\partial_w J$ and $\partial_{ww} J$ are the abbreviations of $\frac{\partial J}{\partial w}$ and $\frac{\partial^2 J}{\partial w^2}$ respectively. Then by substituting Eq.(23) and the fixed point Eq.(17) into Eq.(25), we

obtain

$$0 = \frac{\partial P}{\partial Q} Q_t \iota_t + \frac{\partial P}{\partial A} A \mu^A + \left(\frac{\partial P}{\partial A} \right)^2 \left(\frac{I^x}{P} + \frac{Q \Theta}{I} \right) (A \sigma^A)^2 + \frac{1}{2} \frac{\partial^2 P}{\partial A^2} (A \sigma^A)^2 + \left(\frac{\rho}{\Theta} - c + I^n \right) P, \quad (26)$$

where

$$I = \int_{\Sigma} \frac{\partial_w J}{\partial_{ww} J} m dw, \quad I^x = \frac{A}{I} \left(\frac{1-\alpha}{r-c} \right)^{\frac{1}{\alpha}} \int_{\Sigma} U m dw, \quad I^n = \frac{1}{I} \int_{\Sigma} \frac{\partial \Psi}{\partial n} \frac{\partial_w J}{\partial_{ww} J} m dw. \quad (27)$$

The resulting pricing equation Eq.(26) can be considered as a Black-Scholes-type partial differential equation (PDE) with the following differences.¹¹ First, the “theta” term in Black-Scholes equation reflecting the variation of the derivative value over time is absent in Eq.(26). Instead, the term $\frac{\partial P}{\partial Q} Q_t$ captures the expected inflation from token issuance. Second, since A_t , the fundamental productivity that drives token price, is not tradable, the coefficient of $\frac{\partial P}{\partial A}$ is $A \mu^A$ rather than zero.¹² Third, the additional third term on the RHS originally comes from the risk term in the F.O.C. as Eq.(24) shows and features the price change risk from holding tokens. Moreover, there is a “flow” term, $(\frac{\rho}{\Theta} - c + I^n)P$, that reflects the excess gain from staking rewards offsetting the staking cost and convenience loss. Note that $\frac{\partial \Psi}{\partial n}$ is negative and represents the marginal decrease in transaction costs. Therefore, I^n is typically negative.

Eq.(26) is an partial differential equation for $P(A_t, Q_t)$, which is difficult to solve in general. Reconsidering the market clearing condition and the definition of Q_t , we find that $P(A_t, Q_t)Q_t = \widehat{x_t + l_t}$. It is actually an alternative form of Eq.(14). $\widehat{x_t + l_t}$ represents the aggregate wealth allocated to the platform and is independent with Q_t , the aggregate amount of tokens, since all the relevant variables are endogenous from A_t . Substituting the preceding equation into Eq.(26) and calculate the partial differentials, we will derive an ordinary differential equation (ODE). Moreover, based on the relationship between platform productivity and the allocated value, we provide two boundary conditions for solving this

¹¹The risk free rate of numeraire is normalized to zero.

¹²If the fundamental productivity is tradable, the coefficient of $\frac{\partial P}{\partial A}$ should be $r^f A$, where r^f is the risk free rate and is set to be zero in our model.

ODE. Proposition 3 concludes the above results.

Proposition 3. Token price and dynamic. P_t is separable with the representation:

$$P_t = P(A_t, Q_t) = \frac{1}{Q_t} V(A_t), \quad (28)$$

where $V(A_t)$ captures the aggregate wealth allocated to the platform, and satisfies the ODE:

$$\begin{aligned} 0 = & V'(A_t) A_t \mu^A + V'(A_t)^2 \left(\frac{I^x(A_t)}{V(A_t)} + \frac{\Theta(A_t)}{I(A_t)} \right) (A_t \sigma^A)^2 + \frac{1}{2} V''(A_t) (A_t \sigma^A)^2 \\ & + \left(\frac{\rho_t}{\Theta(A_t)} - c_t + I^n(A_t) - \iota_t \right) V(A_t), \end{aligned} \quad (29)$$

where I , I^x and I^n are denoted as Eq.(27). The ODE is solved with a lower boundary condition,

$$\lim_{A_t \rightarrow 0} V(A_t) = 0, \quad (30)$$

and an upper boundary condition,

$$\lim_{A_t \rightarrow \infty} V(A_t) = \int_W w_t m(t, w_t) dw_t. \quad (31)$$

The drift μ_t and diffusion σ_t in the token price dynamic process Eq.(2) are given by

$$\begin{aligned} \mu_t &= \frac{V'(A_t)}{V(A_t)} A_t \mu^A + \frac{1}{2} \frac{V''(A_t)}{V(A_t)} (A_t \sigma^A)^2 - \iota_t, \\ \sigma_t &= \frac{V'(A_t)}{V(A_t)} A_t \sigma^A. \end{aligned} \quad (32)$$

The economic implications of Eq.(29) are similar to our previous discussion of Eq.(26). Note that the subset of the whole crowd W , Σ , is determined by agents' trade-offs between transaction convenience and staking reward. Therefore, Σ is related to A_t that leads to I , I^x and I^n to be functions of A_t . In addition, A_t the fixed point problem Eq.(17) is substituted, the equilibrium reward rate r_t is replaced by $\frac{\rho_t}{\Theta(A_t)}$. As for the boundary conditions, the lower boundary corresponds to the case that the platform is unproductive and thus attracts no users (staking rewards would be zero too), while the upper boundary represents that when

A_t tends to infinity, the entire population allocate their wealth to the platform. See proof and details about the numerical solution in Appendix A.3.¹³ Agents expect token price to appreciate when they expect higher future productivity. Eq.(32) also implies that expected inflation is reflected in the depreciation of token price.

As Proposition 3 shows, under a fixed inflation rate, in equilibrium, the expected price drift, μ_t , and staking ratio, Θ_t , are both functions of platform productivity, A_t . Figure 3 shows the joint dynamic of these two variables. In general, greater staking ratio relates to higher expected price appreciation. Specifically, μ_t exhibits a roughly convex increase in $(0, 1)$ with an additional lift when staking ratio is sufficiently high.

There are two main economic driving forces. The first is the direct effect of productivity A_t . On the one hand, μ_t declines in A_t . As A_t grows, the pool of agents not entering the economy shrinks, thus the potential future price appreciation is reduced, which generates the similar user-base stabilizing effect of tokens as Cong et al. (2021d). On the other hand, Θ_t also declines in A_t , since higher A_t results in larger transaction convenience. Therefore, the joint dynamic of μ_t and Θ_t represents a positive relationship. This mechanism also explains the steeper slope when staking ratio is low. It reflects the case when A_t is so high that most wealth has already allocated in the platform.

The second mechanism helps explain the additional lift of μ_t when staking ratio is high. When A_t is sufficiently low, it is better for agents to stake and earn reward than to obtain transaction convenience. Almost all the tokens are staked, thus less tokens are in circulation to clear any given amount of business transactions on the platforms where tokens are media of exchange, especially comparing with the economy without a staking choice. Therefore, the value of the tokens much appreciate for any fixed velocity of token.¹⁴ From another perspective, staking is equivalent to offering agents an alternative option when earning transaction convenience is not very beneficial. Thus when A_t is low, the platform has already attracts more adoption than the economy without staking mechanism. Then as A_t increases from a

¹³Numerically, we choose a sufficient small ϵ and correspondingly obtain a very large $A(\epsilon)$, so that $V(A(\epsilon)) = \int_W w_t m(t, w_t) dw_t - \epsilon$ approximately. In addition, we decrease ϵ until the new resulting solution is numerically indistinguishable.

¹⁴In practice, this may partially explain the regular phenomenon that when a stakable asset is first launched on an exchange, its price will rise significantly, even though its productivity is quite low as an incipient asset.

low level, the potential future price appreciation reduces with a steeper slope. Appendix B.1 documents some extended discussions on the joint dynamic of staking ratio and price drift, including an intuitive explanation for non-monotonic increase as Figure 3 shows.

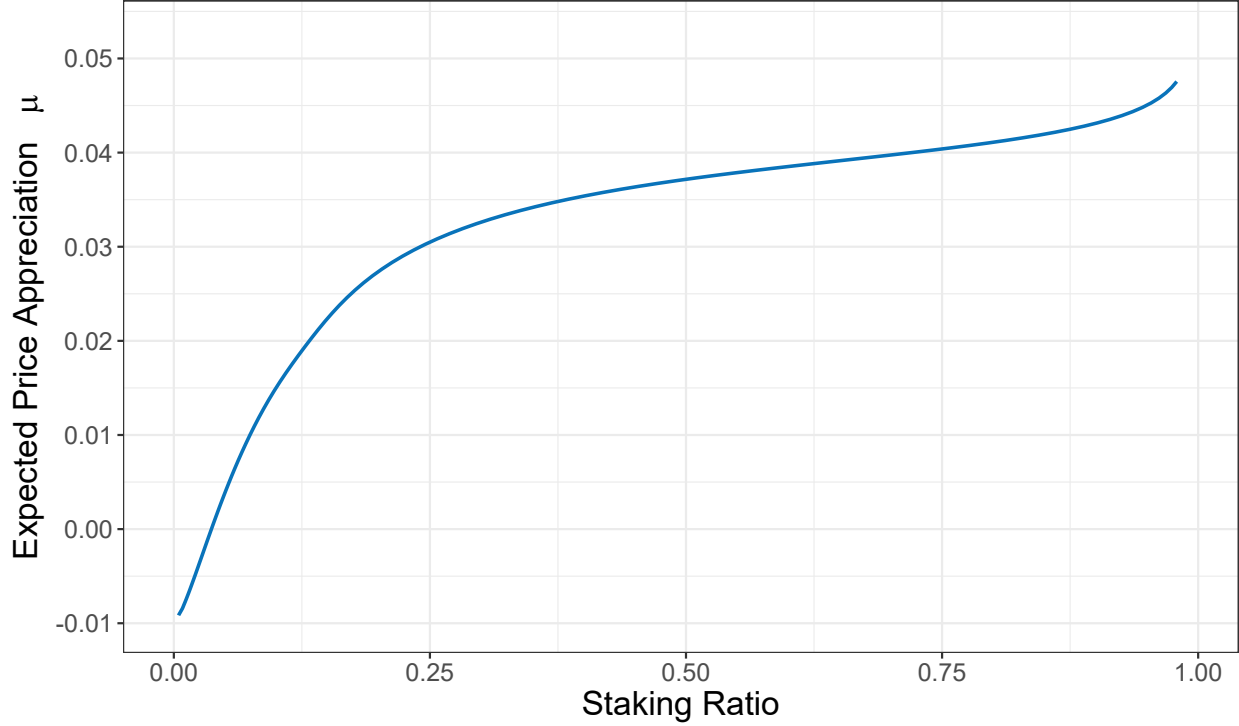


Figure 3: Staking ratio and price dynamics.

This graph shows the relationship between the system staking ratio Θ_t and the drift term of the token price μ_t . As this graph shows, greater staking ratio relates to higher expected price appreciation. Specifically, μ_t exhibits a roughly convex increase in $(0, 1)$ with an additional lift when staking ratio is sufficiently high.

4.3 Token Excess Returns & UIP Violation

For each agent, whether she holds staked tokens or non-staked tokens, she take on the return and risk of token price fluctuation, but loses the convenience of numeraire. Denote the expected financial excess return of staked tokens over the numeraire as λ_t , then

$$\lambda_t \equiv E_t [dP_t + r^{staked\ token}] = \mu_t + r_t - c_t. \quad (33)$$

Proposition 4. *Token excess return.* *For any agent with any positive wealth, her optimal*

aggregate value of token holding, q_t^* , satisfies

$$\lambda_t = -\frac{\partial \Psi}{\partial n_t} - \frac{q_t^* \sigma_t^2 \partial_{ww} J}{\partial_w J} + \frac{\min \{0, MU_x^* - MU_l^*\}}{\partial_w J}, \quad (34)$$

where $-\frac{\partial \Psi}{\partial n_t} > 0$ is the marginal convenience yield of holding numeraire, and MU_l^* and MU_x^* are marginal utility of staked and tradable tokens respectively when agent's controls are optimized.

Eq.(34) has been rearranged so that the left-hand side contains only λ_t , which shows that there are general predictable excess returns that arise as a compensation for convenience loss. Especially, staked token is compensated with staking rewards as financial returns for the loss of transaction convenience.

From another perspective, this is closely related to the uncovered interest rate parity (UIP) in the foreign exchange market. If we treat token as cryptocurrency, then its price is corresponding to the concept of exchange rate, while the staking reward rate corresponds to the concept of interest rate. UIP implies that the expected returns on default-free deposits across currencies are equalized, and thus the expected excess return λ_t should be zero.¹⁵ However, Eq.(34) shows that the uncovered interest parity does not hold, since there are predictable excess returns that arise as a compensation for convenience loss. When the convenience of numeraire increases, staked token is compensated with higher financial return. This interpretation of UIP violation shares similar ideas with Valchev (2020)'s explanation of the UIP puzzle in classical asset types. The second term on the R.H.S represents the impact of volatility risk. This correlates with the conclusions drawn from literatures that use term structure models (e.g., Bansal, 1997; Lustig et al., 2019), where the difference between domestic and foreign bond risk premia, expressed in domestic currency, is determined by volatility difference of the permanent components of the stochastic discount factors. The remaining term on the R.H.S. represents the trade-off between staking and non-staking.

There are two further insights for Eq.(34). First, the excess return λ_t is a system state that

¹⁵In the uncovered interest rate parity among currencies, $\lambda_t = E_t \left[dS_t + i_t^{foreign} - i_t^{local} \right]$, where S_t is the log exchange rate (foreign currency units per unit of local currency). The corresponding terms in Eq.(33) of S_t , $i_t^{foreign}$ and i_t^{local} are respectively dP_t , $r_t - c_t$ and the numeraire risk free rate (normalized to zero).

can be considered exogenous when a single agent makes decision. Second, the convenience of numeraire is a relative concept, which in fact reflects the difference in convenience between numeraire and token.

The first fact suggests that Eq.(34) also implies a trade-off between staking and non-staking by the agent. In other words, the staking reward ($r_t - c_t$) should actually be considered as a compensation for the loss of transaction convenience. More discussion is provided in Appendix A.4. The second fact gives two important corollaries. First, even based on the same numeraire, the expected process return can be different for different tokens, since the convenience of numeraire is a relative concept. Second, not only can we use a currency such as USD as numeraire, we can also use any of the cryptocurrencies as numeraire. Therefore, within the cryptocurrency market, UIP is Violation too.

5 Empirical Analyses

In this section, we test three predictions of our model in empirical: (i) Staking reward rate r_t affects agents' staking choice and thus the overall staking ratio Θ_t . (ii) Staking ratio Θ_t predicts price dynamics and token returns. (iii) Uncovered interest rate parity does not work in the cryptocurrency market.

5.1 Linking Staking Reward Rate to Aggregate Staking

Proposition 2 predicts that a higher staking reward corresponds to a higher system staking ratio in equilibrium. To test this implication empirically, we calculate the daily average of aggregate staking reward ratio and staking ratio for each token over its entire sample period. To make it reasonable for comparisons among different tokens, we use the concept of relative staking reward, i.e., the total amount of tokens used as staking reward divided by the total amount of issued tokens, which is denoted as ρ in our model. Figure 4 plots the relationship between staking reward and staking ratio, in which each token generates one scatter point. The grey dashed line shows the linear fit of the scatters. It has a positive slope, which indicates that the reward has a positive relationship with the staking ratio. As

the figure shows, most scatters are in the region where the relative reward is less than 0.15, while the other tokens might be strong influential points or “outliers”. After removing these points, the linear smooth as the blue dashed line shows that the positive correlation still holds, with an even larger slope. This visualizes the implication as Proposition 2 discusses. Since the proposition is based on the equilibrium case, Figure 4 implicitly illustrates that averaging over the time series roughly conforms to the equilibrium. We also visualize the relationship with shorter data coverage (up to Oct. 2020) as Figure 4 (b) shows. Although there are fewer stakable tokens in the earlier period, the significant positive correlation between the staking ratio and staking reward ratio still exists, which suggests the relationship is robust in the sample period.

We further test the implication by panel regression. We take staking ratio, $\Theta_{i,t}$, as a dependent variable, and the (relative) aggregate staking reward ratio, $\rho_{i,t}$, as the main explanatory variable. The variables are selected at the same period. Table 2 reports the results. As column (1) shows, the estimated coefficient of $\rho_{i,t}$ is positive and significant at a 1% level. The value of estimation implies if the aggregate reward increases by 0.1 units, then the staking ratio will increase by 0.080 (8.0%). We also test with control variables including market value and price volatility. Their potential effect is that crypto assets with greater market value are more trustworthy, making it more attractive for agents to lock in their wealth. The results show a significant impact of control variables, but neither of these two control variables affects the main effects of the staking reward ratio. We also run the regression on data with different periods and fixed effects.¹⁶ As Table 2 reports, the positive correlation between staking reward and staking ratio is robust.

The previous test focuses on static properties in equilibrium, while for agents’ perspective, the main concern is the reward rate they can earn and the resulting portfolio allocation. As Proposition 1 shows, a higher reward rate r_t will lead agents to stake more. Theoretically, the resulting high staking ratio will decrease the reward rate and evolves to equilibrium, while in practice, this process takes time so that Proposition 1 generates a predictable hypothesis on the staking ratio. Empirically, we test the prediction by panel regressions as Table 3 reports.

¹⁶We fixed time effect but not the crypto-specific effect in this test, since the more significant effect in the time series, in practice, is the endogenous decreasing of the reward (similar to Bitcoin Halving).

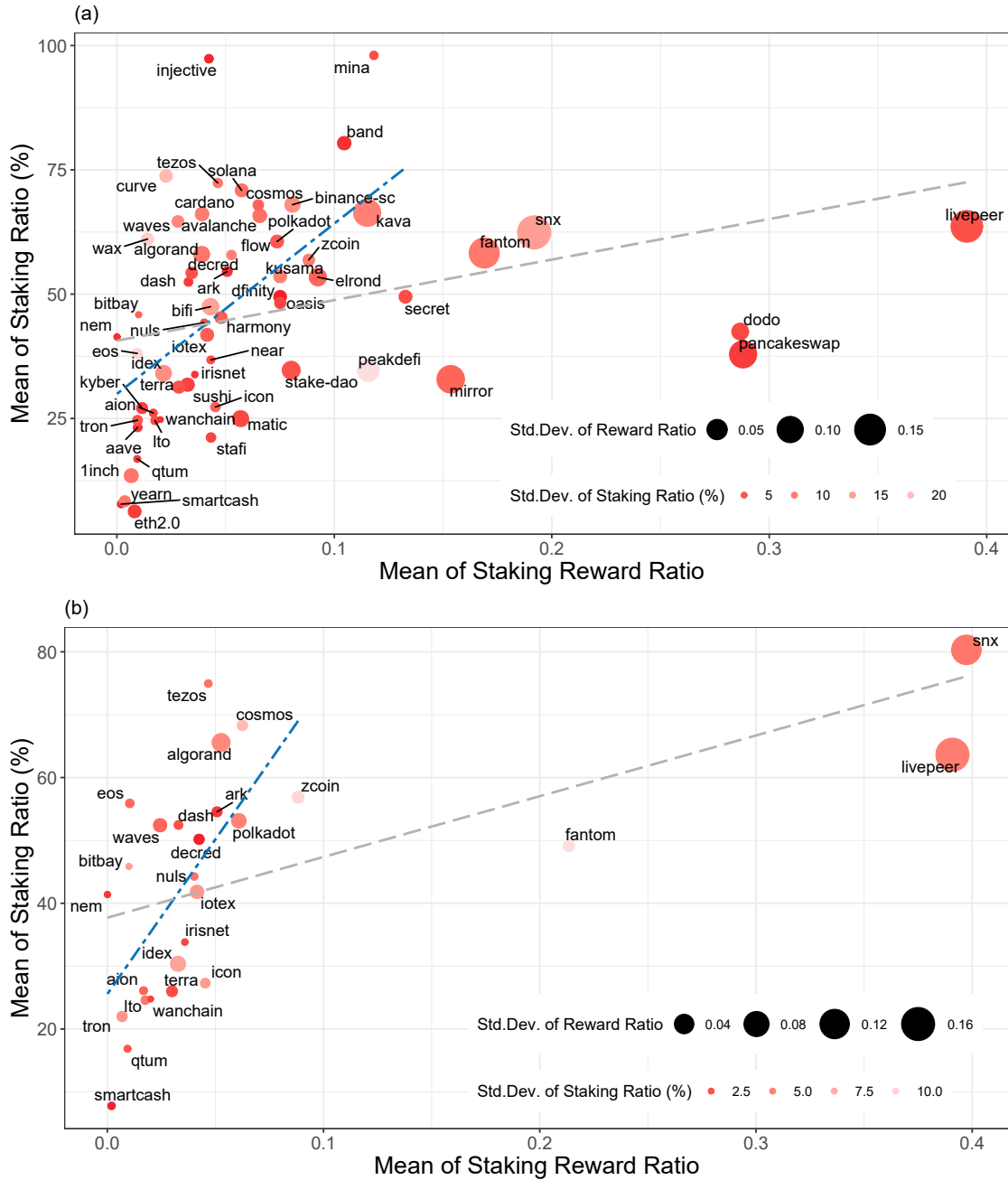


Figure 4: Staking ratio versus staking reward.

This figure plots the relationship between staking ratio Θ_t and staking reward ratio ρ_t . In panel (a), for each token, we calculate its mean staking ratio and reward over the entire time interval (up to Feb. 2022) and then generate one point. The grey dashed line is the linear regression of all scattered points, which shows a positive correlation between the two variables. After removing the influential points with large rewards, the linear regression is still upward sloping and even steeper as the blue dashed line shows. This plot visualizes the results of Proposition 2, i.e., a higher aggregate staking reward ratio leads to a higher system staking ratio. In addition, the size and color of the points indicate the standard deviations of the reward ratio and staking ratio respectively. In panel (b), we do the same thing with shorter data coverage (up to Oct. 2020). Although there are fewer tokens, the main relationship between the staking ratio and the staking reward ratio still holds, which implies robustness in the different sample periods.

Table 2: Staking ratio with respect to the staking reward ratio.

This table tests the relationship between staking ratio, $\Theta_{i,t}$, and the aggregate staking reward ratio, $\rho_{i,t}$, in the same period. The coefficient of $\rho_{i,t}$ is significantly positive, which implies higher staking reward results in a higher staking ratio as Proposition 2 shows. The effect is robust under multiple tests, including controls (market value and volatility), time fixed effects and different horizons (weekly, 14-day and 30-day).

***, **, * indicate statistical significance at the 1%, 5% and 10% respectively.

	<i>StakingRatio_{i,t}</i>					
	7-day				14-day	30-day
	(1)	(2)	(3)	(4)	(5)	(6)
$\rho_{i,t}$	0.800*** (19.420)	0.843*** (20.477)	0.838*** (20.250)	0.865*** (20.334)	0.904*** (14.589)	0.922*** (9.429)
$\frac{1}{100} \log(\text{Cap})_{i,t}$		1.165*** (8.275)	1.187*** (8.379)	1.328*** (7.358)	1.369*** (5.252)	1.507*** (3.804)
Volatility _{i,t}			0.140 (1.395)	0.392*** (3.218)	0.549*** (2.664)	0.816** (2.064)
Fixed Effects						
Time				Y	Y	Y
R ²	0.088	0.103	0.104	0.108	0.117	0.121

Panel A in Table 3 focuses on the cross-sectional situation where, in practice, agents face multiple tokens with different staking reward rates at the same time. We use the reward rate in the previous period, $r_{i,t-1}$, as the main independent variable, and the change in staking ratio, $\Delta\Theta_{i,t} = \Theta_{i,t} - \Theta_{i,t-1}$, as the dependent variable. The estimated coefficients of reward rate are all significantly positive, which implies larger reward rate predicts a positive change of staking ratio. For example, as column (6) shows, if the annual reward rate increases by 1%, then the overall staking ratio will increase by 0.016% in the following week. This is a large effect considering the size of the time window and the magnitude of the change in the rewards rate in the staking economy. Market cap and token price volatility are used as control variables, since they may be related to the platform's user base and risk, thus affecting the overall staking ratio. Fixed effects are also considered. In addition, we also run the test with the staking ratio $\Theta_{i,t}$ as the dependent variable. The estimated coefficient of reward rate is still positive and significant.

Panel B in Table 3 replaces the main independent variable with the change in reward rate in the previous time span, $\Delta r_{i,t-1} = r_{i,t-1} - r_{i,t-2}$. This will erase the cross sectional

difference in the size of the reward rate of tokens, and focus on time series effects. In practice, agents can also adjust their staking choices for a given token based on the change in reward rate over time. We regress the change in staking ratio on the change of reward rate of the previous period. All the regressions in Panel B report the positively estimated coefficient of $\Delta r_{i,t-1}$ and, in particular, these results are significant at least at the 10% level in the regressions on weekly data. The significantly positive coefficient proves that people tend to stake more as the reward rate rises. For example, column (6) implies that for a certain token, if there is a 1% increase in its reward rate in the previous week, then its overall staking ratio will increase by 0.033% in the following week. Market value and volatility are also considered as controls. Note that in columns (7) and (8), the estimation is still positive but not significant. This can be explained by the fact that the super-long horizon contains more changes and perturbations, even including the dynamic adjustment process between the reward rate and the staking ratio section 4.1 discusses.

5.2 Equilibrium Staking Ratio and Token Price Dynamics

Our model shows that the staking ratio positively predicts token price changes. To test this prediction, we first calculate the log token price change in each period for each token, $r_{price_{i,t}} = \log(\frac{P_{i,t}}{P_{i,t-1}})$. We regress $r_{price_{i,t}}$ on the staking ratio in the previous period. As many papers have discussed, the market and market value factors have an important impact on price change, which also holds for the cryptocurrency market. Therefore, we add the current period market factor and the previous period log token market cap to the regression as controls. The calculated data for r_{MKT} is shared by Cong et al. (2021a), and the original data is collected from *CoinMarketCap.com*.

Table 3: Staking ratio with respect to the staking reward rate.

This table presents the analyses of how people’s staking choice is affected by the reward rate. In Panel A, we use the reward rate of the previous period, $r_{i,t-1}$ as independent. The regressions show significantly positive coefficient, which implies that larger reward rates predict the positive change of staking ratio $\Delta StakingRatio_{i,t}$. This effect is robust with controls (market value and volatility), fixed effects (crypto-specific effect, time effect and two-way effect), and in different horizons (weekly, 14-day and 30-day). In Panel B, we replace $r_{i,t-1}$ by the change of reward rate of the previous period, $\Delta r_{i,t-1}$, as the independent variable. The significantly positive coefficient implies that people tend to stake more as the reward rate rises.

***, **, * indicate statistical significance at the 1%, 5% and 10% respectively.

	$\Delta StakingRatio_{i,t}$							
	7-day						14-day	30-day
<i>Panel A</i>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$r_{i,t-1}$	0.008*** (2.868)	0.008*** (2.780)	0.008*** (2.754)	0.015*** (2.997)	0.008*** (2.973)	0.016*** (3.143)	0.026*** (2.600)	0.043* (1.930)
$\frac{1}{100} \log(\text{Cap})_{i,t-1}$		-0.008 (-0.454)	-0.008 (-0.422)	-0.043 (-1.115)	0.009 (0.409)	-0.012 (-0.169)	-0.037 (-0.260)	-0.012 (-0.037)
Volatility $_{i,t-1}$			0.003 (0.266)	0.005 (0.360)	0.012 (0.824)	0.014 (0.819)	0.002 (0.058)	0.046 (0.426)
Fixed Effects								
Crypto				Y		Y	Y	Y
Time					Y	Y	Y	Y
R ²	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005
<i>Panel B</i>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta r_{i,t-1}$	0.022* (1.654)	0.022* (1.678)	0.022* (1.682)	0.026* (1.898)	0.030** (2.155)	0.033** (2.375)	0.017 (0.884)	0.010 (0.369)
$\frac{1}{100} \log(\text{Cap})_{i,t-1}$		-0.016 (-0.884)	-0.015 (-0.821)	-0.067* (-1.769)	-0.004 (-0.193)	-0.030 (-0.428)	-0.068 (-0.487)	-0.067 (-0.212)
Volatility $_{i,t-1}$			0.006 (0.474)	0.007 (0.486)	0.015 (1.003)	0.014 (0.850)	0.002 (0.051)	0.043 (0.397)
Fixed Effects								
Crypto				Y		Y	Y	Y
Time					Y	Y	Y	Y
R ²	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.000

Table 4 reports the results that the staking ratio predicts price appreciation. The estimated coefficient of staking ratio is significantly positive, which implies that a higher staking ratio predicts larger token price appreciation. As column (3) shows, if the staking ratio of a token increases by 1%, its price will appreciate by 0.221% next week. Considering there is often a large variation in the staking ratio, this effect can have a significant impact on price. This result remains robust and significant with the addition of control variables. Therefore, the effect of price appreciation due to the staking ratio is not explained by the market value effect. In addition, the estimated coefficients of r_{MKT} and log capitalization are consistent with related research.

As Section 4.2 discusses, the key to explaining this effect is that the staking ratio closely affects the number of tokens in circulation on the platform. Since there is always demand for the transaction, when the staking ratio is greater, it means that fewer tokens are used as a medium to clear the market, and therefore the value of the token appreciates more for any given velocity of a token.¹⁷

5.3 UIP Violation

Uncovered Interest Parity (UIP) plays a central role in exchange rate determination in most models, which implies that the expected exchange rate depreciation offsets any potential gains from interest rates. However, numerous empirical studies have shown that there is a so-called “UIP puzzle” that an increase in the foreign interest rate relative to the local one is associated with an increase in the excess return on the foreign currency over the local currency. As a practical application of UIP violation, the carry trade strategy is widely used in the exchange market.

In the staking cryptocurrency market, we can also think about similar issues. Staking reward rate can be considered as interest rate, while token price can be viewed in the context of the exchange rate. Then the UIP implies that

$$E_t [\log P_{t+1} - \log P_t] = r_t^f - (r_t - c_t), \quad (35)$$

¹⁷Token velocity is immaterial in our continuous-time formulation. In general, token staking and lockup can be viewed as reducing their velocity in the ecosystem.

Table 4: Staking ratio and token prices.

This table presents the analyses of how the staking ratio predicts token price appreciation. The main independent is the staking ratio of the previous period, $StakingRatio_{i,t-1}$. The dependent $r_{price_{i,t}}$ is the log price change. The results show that the coefficient is significantly positive, which implies that a higher staking ratio will predict higher token price appreciation. Considering that there exists market effect and market value effect in the cryptocurrency market, we also add the market price return r_{MKT_t} and the market cap term $\log(Cap)_{i,t-1}$ as controls. After adding these controls, the estimated coefficient of staking ratio is still significant. We also do the test in different horizons and with fixed effects to show the robustness of the results.

***, **, * indicate statistical significance at the 1%, 5% and 10% respectively.

	$r_{price_{i,t}}$				
	7-day			14-day	30-day
	(1)	(2)	(3)	(4)	(5)
$StakingRatio_{i,t-1}$	0.193** (2.094)	0.183** (2.148)	0.221*** (2.621)	0.447*** (2.830)	0.720* (1.808)
r_{MKT_t}		0.821*** (15.863)	0.790*** (15.396)	0.617*** (9.226)	0.854*** (8.000)
$\log(Cap)_{i,t-1}$			-0.956*** (-6.191)	-1.487*** (-4.973)	-3.457*** (-4.139)
Fixed Effects					
Crypto	Y	Y	Y	Y	Y
R ²	0.003	0.150	0.171	0.159	0.300

where r_t^f is the local interest rate at time t .

In our model, the uncovered interest rate parity does not work in the cryptocurrency market. To empirically test if UIP works, we use the equation with some version of the original regression specification of Fama (1984) as follows:

$$\lambda_{i,t+1} = \alpha + \beta(r_t^f - r_{i,t} + c_{i,t}) + \epsilon_{i,t+1}, \quad (36)$$

where $\lambda_{i,t} = \log P_{i,t+1} - \log P_{i,t} + (r_{i,t} - c_{i,t}) - r_t^f$,

where i represents cryptocurrency i . Under UIP, $\beta = 0$, i.e. the excess return λ_t is not forecastable by the current interest rate difference. On the contrary, numerous empirical researches have found that β is not equal to 0, and even find that $\beta < 0$ so that higher interest rates are associated with higher excess returns. For the local currency, we examine in turn each asset in our sample as well as the US dollar, Bitcoin, and Ethereum. We also examine different time horizons as Valchev (2020) does.

Table 5 reports the regression results. In each row, we report the result of a specific asset as a local currency, i.e., the exchange rate of each token is converted to the price denominated in such asset. All the results show a significantly negative estimation of β , which violates the UIP. Moreover, $\beta < 0$ and is close to 1, which implies that a higher interest rate will predict a positive appreciation of the exchange rate. This leads to potential arbitrage opportunities. The regression results with different tokens as the local currency all demonstrate the UIP violation, which implies this phenomenon exists not only among the stakable tokens in our sample, but also exists when comparing with traditional currencies and mainstream non-stakable cryptocurrencies.

6 Crypto Carry

As an extension of UIP violation, we test the predictability of crypto carry to token excess return and the performance of the crypto carry trade portfolio.

6.1 Carry in Other Asset Classes

Carry trades, which go long in baskets of currencies with high interest rates and short in baskets of currencies with low interest rates, have been shown to obtain high Sharpe ratios. The portfolio performance, the predictability of carry to excess returns and the possible explanation has been widely studied (e.g., Lustig et al., 2014; Bakshi and Panayotov, 2013; Burnside et al., 2011; Menkhoff et al., 2012; Kojen et al., 2018; Daniel et al., 2017).

Carry strategies are profitable for a host of different asset classes, including global equities, global bonds, commodities, US Treasuries, credit, and options. But crypto assets may differ from any of the traditional asset classes in terms of characteristics. On the other hand, the characteristics of the cryptocurrency markets operation may also generate new features and arbitrage opportunities (e.g., Makarov and Schoar, 2020). Therefore, carry and related contents on crypto assets may exhibit unique properties, which is discussed in recent literature (e.g., Franz and Valentin, 2020).

Table 5: Test on the UIP violation.

This table reports the panel regression results of UIP test. The regression model is shown as Eq.(36). In each row, we use a different asset as local currency and report the estimated coefficients of β with different data horizons. The estimated coefficient of β and its t-statistic is reported. All the results shows significantly negative estimation of β , which prove that UIP violates. Moreover, $\beta < 0$ implies that higher interest rate will predict positive appreciation of exchange rate. The table also shows the results are consistent with the relevant research results of classic assets, and are robust among currencies and cryptocurrencies.

Local	Horizon: 7-day			Horizon: 30-day			Local	Horizon: 7-day			Horizon: 30-day		
Currency	Coef., β	t-statistic	R^2	Coef., β	t-statistic	R^2	Currency	Coef., β	t-statistic	R^2	Coef., β	t-statistic	R^2
<i>Currency & mainstream cryptocurrencies.</i>													
US Dollar	-0.98	(-49.40)	0.38	-0.93	(-9.95)	0.11	Ethereum	-0.99	(-59.13)	0.46	-0.95	(-13.52)	0.17
Bitcoin	-0.99	(-59.78)	0.47	-0.96	(-14.04)	0.18							
<i>Cryptocurrencies in our sample.</i>													
linch	-0.98	(-34.07)	0.48	-0.83	(-6.08)	0.14	kyber	-1.01	(-23.59)	0.33	-0.99	(-4.62)	0.10
aave	-1.01	(-39.36)	0.41	-0.99	(-8.18)	0.14	livepeer	-1.01	(-25.14)	0.34	-0.85	(-4.44)	0.07
aion	-0.97	(-25.07)	0.39	-0.80	(-5.29)	0.12	lto	-1.05	(-18.90)	0.31	-0.99	(-5.92)	0.18
algorand	-0.98	(-54.28)	0.43	-0.94	(-14.02)	0.18	matic	-0.96	(-31.18)	0.26	-0.90	(-6.27)	0.06
ark	-0.99	(-35.74)	0.50	-0.91	(-9.35)	0.24	mina	-0.99	(-32.38)	0.42	-0.81	(-6.15)	0.11
avalanche	-1.03	(-36.32)	0.36	-1.12	(-8.70)	0.13	mirror	-0.98	(-34.23)	0.46	-0.97	(-6.51)	0.15
band	-1.03	(-43.11)	0.40	-1.02	(-10.87)	0.16	near	-1.00	(-41.01)	0.41	-1.01	(-9.61)	0.14
bifi	-0.98	(-34.75)	0.39	-0.92	(-6.84)	0.10	nem	-0.97	(-31.84)	0.58	-0.79	(-8.08)	0.29
binance-sc	-0.99	(-45.76)	0.52	-0.92	(-8.61)	0.15	neo	-1.00	(-33.61)	0.46	-0.89	(-8.88)	0.21
bitbay	-0.95	(-7.91)	0.12	-1.36	(-3.64)	0.34	nuls	-1.01	(-28.53)	0.42	-0.96	(-6.89)	0.17
cardano	-0.98	(-44.57)	0.43	-0.99	(-10.53)	0.16	oasis	-0.99	(-33.83)	0.40	-0.91	(-6.89)	0.11
cosmos	-0.99	(-55.54)	0.43	-0.95	(-12.86)	0.16	pancakeswap	-0.99	(-40.05)	0.45	-0.89	(-8.55)	0.14
curve	-1.00	(-39.16)	0.45	-0.93	(-9.65)	0.19	peakdefi	-0.95	(-32.63)	0.34	-0.85	(-7.96)	0.12
dash	-0.98	(-33.19)	0.46	-0.79	(-5.77)	0.11	polkadot	-1.02	(-47.72)	0.45	-1.06	(-11.43)	0.18
decred	-0.99	(-55.98)	0.45	-0.97	(-13.34)	0.17	qtum	-0.98	(-38.12)	0.54	-0.88	(-10.10)	0.28
dfinity	-0.94	(-20.84)	0.42	-0.90	(-3.79)	0.11	secret	-1.02	(-39.21)	0.38	-1.15	(-8.95)	0.13
dodo	-0.91	(-24.55)	0.40	-0.83	(-5.40)	0.11	smartcash	-0.99	(-26.16)	0.38	-0.83	(-6.83)	0.16
elrond	-1.00	(-41.89)	0.42	-1.07	(-9.33)	0.14	snx	-1.02	(-51.35)	0.41	-1.08	(-12.35)	0.15
eos	-0.99	(-51.72)	0.43	-0.91	(-11.93)	0.15	solana	-0.97	(-34.06)	0.37	-0.93	(-7.58)	0.11
eth2.0	-1.01	(-52.72)	0.46	-1.02	(-12.05)	0.17	stafi	-1.00	(-32.80)	0.32	-0.98	(-8.11)	0.12
fantom	-0.99	(-30.47)	0.21	-1.06	(-8.80)	0.09	stake-dao	-0.95	(-16.07)	0.20	-0.83	(-3.07)	0.05
flow	-0.94	(-26.56)	0.41	-0.80	(-5.40)	0.11	sushi	-0.98	(-40.81)	0.45	-0.90	(-9.69)	0.17
harmony	-0.98	(-37.35)	0.32	-0.92	(-6.19)	0.06	terra	-0.99	(-38.65)	0.33	-0.86	(-7.57)	0.08
icon	-0.97	(-31.34)	0.45	-0.85	(-7.25)	0.17	tezos	-1.00	(-58.02)	0.45	-0.94	(-12.96)	0.16
idex	-1.00	(-41.29)	0.36	-0.96	(-9.50)	0.12	tron	-0.98	(-57.66)	0.46	-0.91	(-12.56)	0.15
injective	-0.93	(-24.45)	0.38	-0.86	(-5.36)	0.13	wanchain	-0.98	(-26.99)	0.43	-0.90	(-7.99)	0.23
iotex	-0.98	(-29.51)	0.40	-0.85	(-6.89)	0.14	waves	-0.99	(-52.11)	0.42	-0.95	(-12.31)	0.15
irisnet	-1.00	(-27.14)	0.39	-0.98	(-7.47)	0.18	wax	-0.98	(-31.31)	0.27	-1.02	(-6.17)	0.06
kava	-1.00	(-47.25)	0.41	-0.96	(-10.53)	0.13	yearn	-1.00	(-37.70)	0.35	-0.97	(-8.75)	0.11
kusama	-1.01	(-39.94)	0.35	-1.09	(-11.74)	0.17	zcoin	-0.99	(-24.61)	0.36	-0.87	(-6.72)	0.16

One of the direct corollary to the violation of UIP is the existence of carry. Kojien et al. (2018) define carry as a general concept of any asset. For any asset, carry is defined as its futures return, assuming that price stays the same, i.e.

$$\text{return} \equiv \text{carry} + E(\text{price appreciation}) + \text{unexpected price shock.} \quad (37)$$

For example, the classic definition of currency carry is the local interest rate in the corresponding country. Following Kojien's general definition of carry, we derive crypto carry based on our model as Eq.(38) shows, which is close to the form of currency carry.¹⁸

$$\text{carry}_t \equiv \frac{r_t - c_t - r^f}{1 + r^f}. \quad (38)$$

Table 6 summarizes annualized carry and excess return of all the tokens in our sample. Sample means and standard deviations are reported. We also include the US Dollar as one of the assets for which the carry and excess return are, by definition, equal to zero.

6.2 Crypto Carry Trade Portfolio Returns

Tokens in the asset pool are ordered by their carry in the previous period, and then divided into three groups, i.e. the top $x\%$ of assets, the bottom $x\%$ and the middle group. Then we construct a carry trade portfolio by going long high carry group with equal weight and going short low with equal weight at the end of each week. For long tokens, we also stake them to earn staking reward rate, while for the short assets, we also compensate for the staking reward rate. The choice of x does not affect our observation of the main characteristics of the carry trade portfolio. The portfolio is rebalanced every week.¹⁹ Considering the abnormal fluctuation of token price and staking ratio when a staking project is first launched, our weekly asset pool does not include new staking projects that come out within

¹⁸This formula is derived based on the assumption that covered interest rate parity holds, where Kojien et al. (2018) also claim this assumption when obtaining the formula of currency carry.

¹⁹We also assume that the staking rules allow a one-week stake period. Most stakable tokens do offer such flexible staking options, and our data of reward rate are also selected in the corresponding options. For some rare exceptions, we can assume the existence of some derivatives that would enable such an asset allocation. Such derivatives are gradually appearing in practice.

Table 6: Excess return and carry.

Token	Excess Return (%, Annual)		Carry (%, Annual)		Token	Excess Return (%, Annual)		Carry (%, Annual)	
	Mean	Std.dev.	Mean	Std.dev.		Mean	Std.dev.	Mean	Std.dev.
linch	2.67	17.52	2.02	4.38	kyber	7.74	18.19	4.32	2.04
aave	7.83	23.38	4.02	0.86	livepeer	59.79	38.12	58.51	26.72
aion	8.76	16.37	5.57	0.48	lto	10.65	18.69	6.80	0.85
algorand	8.31	21.25	6.04	2.72	matic	32.26	40.29	23.53	12.09
ark	9.55	17.61	8.11	0.50	mina	13.63	22.56	11.75	0.28
avalanche	19.15	37.19	9.99	2.03	mirror	40.07	36.68	43.10	24.83
band	15.37	25.93	12.70	1.51	near	18.59	23.51	11.50	1.20
bifi	10.12	20.00	7.95	3.57	nem	-1.88	13.54	-1.35	0.49
binance-sc	13.50	15.18	11.33	3.31	neo	1.94	15.92	0.82	0.91
bitbay	18.36	77.83	1.06	0.74	nuls	10.50	18.76	8.18	0.40
cardano	9.54	18.32	5.69	1.87	oasis	18.97	25.39	15.25	2.31
cosmos	12.30	19.99	8.91	1.48	pancakeswap	75.05	45.23	71.38	22.63
curve	5.49	21.52	3.03	1.86	peakdefi	43.78	37.06	43.09	17.31
dash	6.43	20.20	5.20	0.71	polkadot	16.07	20.28	11.90	1.39
decred	7.96	15.95	5.90	1.46	qtum	6.52	14.86	4.65	1.32
dfinity	9.26	16.95	14.49	2.32	secret	34.66	30.76	26.69	2.62
dodo	56.18	23.97	65.74	5.00	smartcash	4.05	18.53	1.62	0.28
elrond	26.07	30.28	17.44	7.12	snx	30.47	32.06	24.96	20.11
eos	4.72	16.53	4.39	3.94	solana	14.35	23.15	7.73	2.48
eth2.0	15.84	17.80	11.58	12.21	stafi	24.71	44.53	20.21	2.14
fantom	41.37	63.53	28.12	22.17	stake-dao	24.03	27.56	22.59	9.17
flow	4.49	14.05	9.21	0.71	sushi	8.75	19.58	10.38	4.47
harmony	18.14	28.10	10.31	0.79	terra	16.14	28.80	8.65	3.09
icon	19.60	21.11	16.03	2.45	tezos	6.96	17.90	5.38	0.82
idex	9.28	24.32	5.88	6.18	tron	5.65	16.08	3.36	1.69
injective	1.63	14.44	4.00	0.36	wanchain	10.30	16.13	7.37	0.30
iotex	10.85	22.54	8.50	2.70	waves	7.21	20.22	3.96	1.70
irisnet	15.33	23.31	9.64	0.39	wax	7.23	21.98	2.80	2.06
kava	22.63	27.97	18.93	20.06	yearn	9.55	30.27	4.68	2.79
kusama	20.71	25.96	13.99	0.84	zcoin	18.18	25.12	14.87	3.53
US Dollar	0.00	0.00	0.00	0.00					

a week.

The performance of such crypto carry trade mainly measures the cross-sectional effect. Since we long high carry and short low carry, the portfolio carry is always positive. If the portfolio always achieves positive returns, it means that in the cross-section, assets with higher carry have greater aggregate returns.

The red curve in Figure 5 plots the cumulative return of such carry trade strategy. It shows an overall increasing and large cumulative returns. Especially, in the cryptocurrency market where price volatility is huge, such a strategy performs a relatively smooth growth, which implies the carry premia always exists. For further discussion, we also report two related strategies. The grey line shows the performance of the same carry portfolio but

without staking. That is, for long tokens, we do not stake them, and for the short assets, we also do not compensate for the staking reward rate. The strategy also exhibits increasing cumulative returns, which implies that the carry strategy earns excess returns not only from carry (staking reward) but also from price appreciation. Moreover, the blue line reports the performance of the same carry portfolio but is rebalanced every month. It exhibits less returns than 1W-carry trade. There are two potential explanations. First, as we discussed in our model, the reward rate will decrease with the staking ratio mechanically. Therefore, investors are unable to earn high carry consistently for a long period without timely position adjustments. Second, the reversal of reward rate further influences the staking ratio, which then weakens the effect on price appreciation as Table 4 reports.

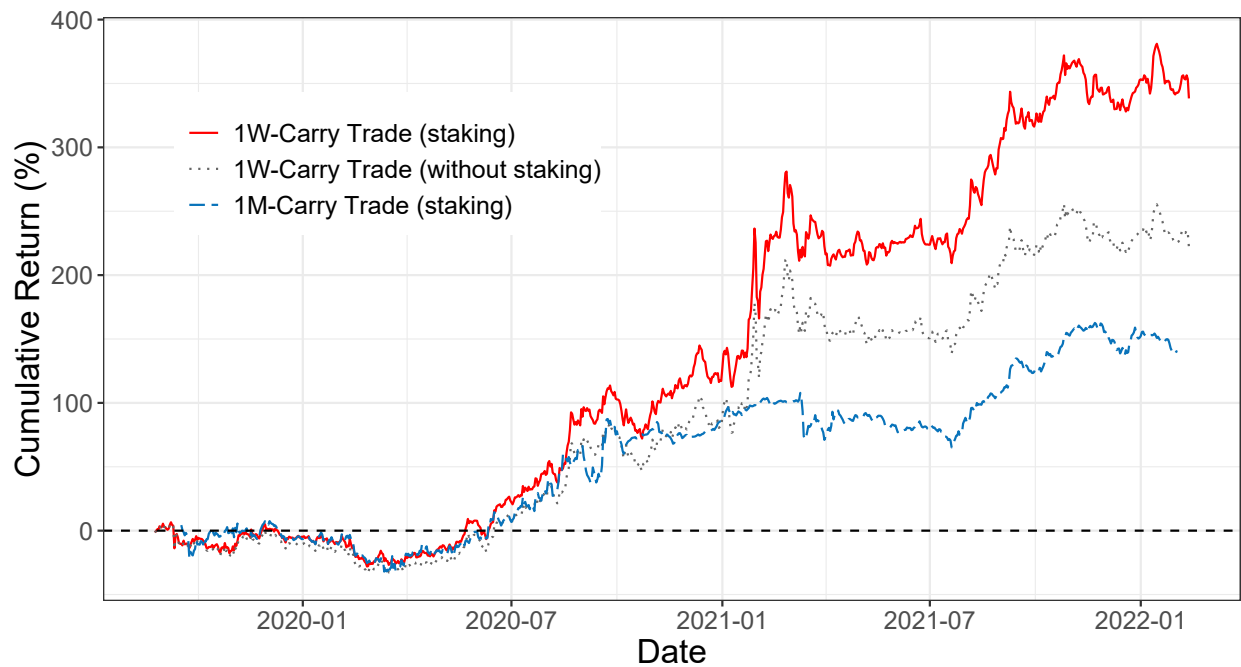


Figure 5: Cumulative returns of long-short carry trade strategies.

This figure shows the cumulative return of long-short carry strategies. The red line is the benchmark strategy. Tokens in the asset pool are ordered by their carry in the previous period. We go long the top $x\%$ high carry tokens with equal weight and short the bottom $x\%$ tokens with equal weight. For long tokens, we also stake them to earn the staking reward rate, while for the short assets, we also compensate for the staking reward rate. The portfolio is rebalanced every week. The choice of x does not affect our observation of the main characteristics. Here we set $x = 50$. Based on the benchmark strategy, the grey curve reports the performance of the strategy without earning or compensating staking rewards, the blue curve shows the performance of the strategy rebalanced every month.

The first row in Table 7 reports statics of the 1-Week carry strategy, including the annualized mean, standard deviations, skewness, kurtosis, maximum drawdown and the

Table 7: Statistics of carry strategies.

This table reports the statistics of three strategies. The first row reports the results of the long-short carry strategy which is corresponding to the red curve in Figure 5, The rows below report long strategies, including equal-weighted benchmark and the strategy that long only top 50% high carry tokens with equal weight. For each strategy, the annualized mean, standard deviations, skewness, kurtosis, maximum drawdown (MDD) and Sharpe ratio are reported.

Strategy	Mean (Annual, %)	St.dev. (Annual, %)	Skewness	Kurtosis	MDD (%)	Sharpe Ratio (Annual)
<i>Long-short Strategy:</i>						
1W-Carry Trade	68.87	42.60	-0.11	6.86	26.23	1.62
<i>Long Strategy:</i>						
EW	95.24	45.78	-1.12	6.59	32.82	2.08
EW High carry	121.08	50.07	-0.89	7.10	32.62	2.42

Sharpe ratio. The results show that the carry strategy has a significantly greater positive return and yields a Sharpe ratio of 1.62. Examining the higher moments of the crypto carry trade return, we find the strong negative skewness associated with the currency carry trade shown by Brunnermeier et al. (2008). Moreover, the carry strategy exhibits excess kurtosis, indicating fat-tailed positive and negative return, which is consistent with Koijen et al. (2018)'s findings for currencies and commodities.

We also report the statistics for equal-weighted strategy in Table 7, i.e. borrow US dollar and go long all tokens in our sample with equal weight. Since the cryptocurrency market is generally in a bull market from 2019 to 2021, the equal-weighted benchmark earns extremely high beta returns and therefore has a very high Sharpe ratio of 2.08, while the carry strategy as a long-short strategy does not earn market returns. As a comparison, we test the strategy that borrows US dollar and buys 50% high carry tokens with equal weight. The order of the tokens is evaluated every week. Such a strategy outperforms the simple equal-weighted portfolio with a Sharpe ratio of 2.42. Figure 6 plots the cumulative returns of these two strategies. The comparison also illustrates the positive correlation of carry to excess returns.

6.3 Excess Return Predicted by Carry

Table 7 suggests that carry is a unique predictor of return. Considering Eq.(33), the predictability may be from two sources: the crypto carry itself, and any price appreciation

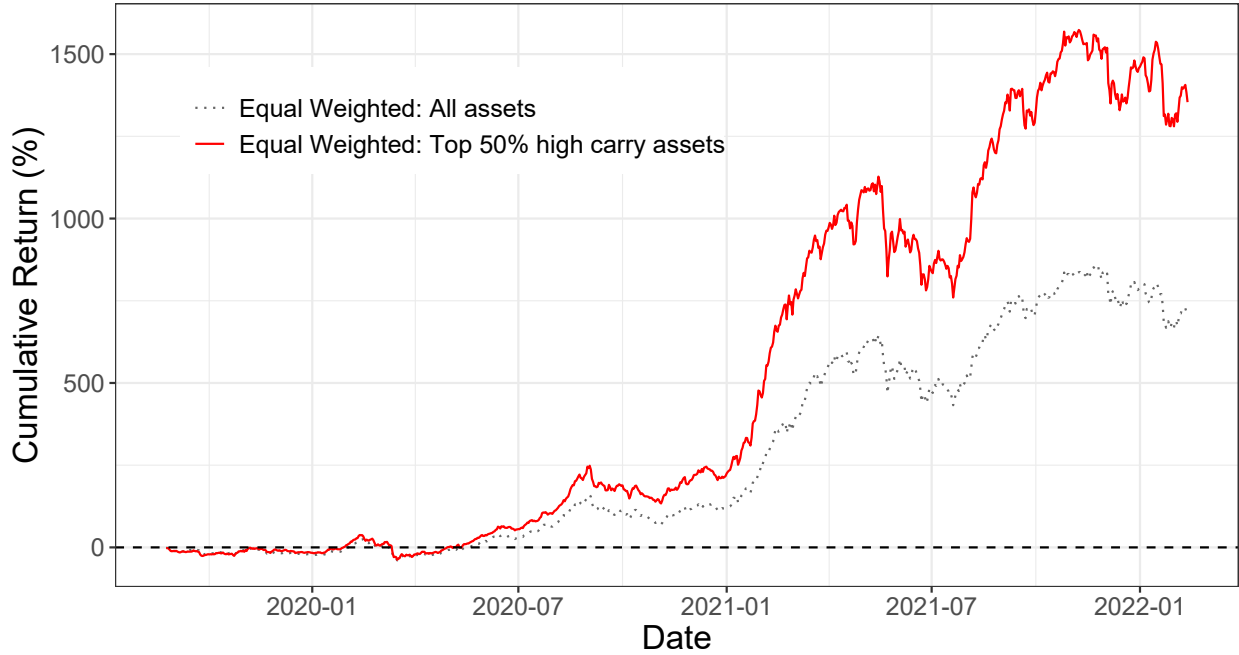


Figure 6: Cumulative returns of long strategies.

This figure shows the cumulative returns of the following two strategies. The grey curve corresponds to the equal-weighted benchmark, i.e. borrow US dollar and long all the tokens with equal weight. The red curve shows the result of the top 50% EW strategy, i.e. borrow US dollar and go long top 50% high carry tokens with equal weight. The order of the tokens is evaluated every week.

that is related to or predicted by carry. To better understand the relationship between carry and expected returns, we follow Kojien et al. (2018) to run the following panel regression:

$$Excess\ Return_{i,t+1} = a_i + b_t + cCarry_{i,t} + \epsilon_{i,t}, \quad (39)$$

where a_i and b_t are crypto and time fixed effect respectively. By Eq.(33), $c = 0$ means the total return is unpredictable, while $c = 1$ suggests that expected return moves one-for-one with carry. If $c \in (0, 1)$, it implies that the market takes back part of the carry, i.e. investors cannot fully earn carry as return.

Table 8 reports the results with and without fixed effects. Weekly and 14-day data are both used for robustness. Without crypto specific and time fixed effects, c represents the total predictability of returns from carry from both its passive and dynamic components. Crypto-specific fixed effects will remove the predictable return component of carry coming from passive exposure to tokens with different unconditional average returns.

The results in Table 8 imply that carry is a strong predictor of expected return. In Columns (1) and (3), without crypto specific fixed effect, the estimated coefficient is around 1, which means that high staking reward rate tokens neither depreciate nor appreciate on average. Hence, investors can earn reward rate differential using carry trade strategy. This is similar to the relevant findings for currency (Fama, 1984; Koijen et al., 2018).

Table 8: Carry and excess returns

This table reports the results from the panel regression of Eq.(39), estimated c and t-statistics are reported. Without crypto and time fixed effects, c represents the total predictability of returns from carry from both its passive and dynamic components. Including crypto specific fixed effect will remove the predictable return component of carry coming from passive exposure to tokens with different unconditional average returns. ***, **, * indicate statistical significance at the 1%, 5% and 10% respectively.

<i>Panel A: 7-day</i>				
	ExcessReturn _{<i>i,t</i>}			
	(1)	(2)	(3)	(4)
Carry _{<i>i,t-1</i>}	0.987*** (37.587)	0.961*** (20.385)	1.004*** (45.090)	0.998*** (24.283)
Fixed Effects				
Crypto		Y		Y
Time			Y	Y
R ²	0.261	0.095	0.347	0.136
<i>Panel B: 30-day</i>				
	ExcessReturn _{<i>i,t</i>}			
	(1)	(2)	(3)	(4)
Carry _{<i>i,t-1</i>}	1.019*** (6.900)	0.625** (2.358)	1.067*** (8.363)	0.691*** (2.939)
Fixed Effects				
Crypto		Y		Y
Time			Y	Y
R ²	0.049	0.006	0.074	0.010

Note that once crypto-specific effect is fixed, as Columns (2) and (4) in Panel B shows, the estimated c is significantly positive but less than 1. This implies the market takes back a fraction of carry. In other words, time series carry predicts less expected return. According to Koijen et al. (2018), this is also found in commodities. When a commodity has a high spot price relative to its futures price, implying a high carry, the spot price tends to depreciate on average, thus lowering the realized return on average below the carry. For crypto in the staking economy, however, a different mechanism may be responsible for this phenomenon. In our model, while a high reward rate leads to a high staking ratio and thus a higher

price appreciation, there is a downward adjustment effect of the reward rate in the time series. As the sum of carry (approximately equal to reward rate) and price appreciation, the excess return is then influenced by the adjustment. Comparing the results of weekly data with those of 30-day data, the downward adjustment effect is also magnified when the time window becomes larger, and thus the estimated c decreases in Columns (2) and (4) of Panel B. This also explains why there is such a difference in the results with and without fixed cross-sectional effects. In the case of commodity, the estimate of c is significantly smaller than 0 regardless of the fixed effect.

Overall, crypto carry exhibits characteristics partly similar to currencies and partly similar to commodities, rationalized by the mechanism of staking itself.

7 Conclusion

In addition to offering a convenience yield for transactions in digital networks, tokens are frequently staked (and slashed) for base-layer consensus generation or for incentivizing economic activities in DeFi protocols and platform development, and consequently earn rewards akin to deposit interests. To analyze the economics of staking, we build a continuous-time model of a token-based economy where agents endogenously allocate wealth on and off a digital platform and use tokens either to earn rewards or to transact. We solve the mean field game with stochastic controls and show that the equilibrium staking ratio is a fundamental variable linking staking to the endogenous reward rate and token price. The model rationalizes violations of the uncovered interest rate parity and significant crypto carry premia that we empirically document. For example, a strategy buying high carry tokens and shorting low carry tokens yields a Sharpe ratio of 1.6, which can be attributed to transaction convenience in certain digital networks. We relate cryptocurrencies to other major asset classes such as currencies and commodities and verify model implications in the data. In particular, the staking ratio resembles liquidity and market depth since a high staking ratio leads to a lower amount of available tokens available for trade. Furthermore, the staking ratio is proportional to the reward rates in the cross-section but negatively correlated to

reward rates in the time series; it positively predicts the returns of cryptocurrencies.

It is worth pointing out that the framework can be used to understand long-run transitions of many platform tokens to security tokens as defined in Cong and Xiao (2021) and Cong et al. (2021a). For example, DeFi projects increasingly lock up both native and non-native tokens whose transaction usage significantly declines. Instead, these tokens entitle the stakers to cash flows from fees (and subsidies in the short run), which adds a security token dimension. Optimally designing such a transition and understanding the implications of staking with multiple tokens constitute interesting future research.

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Appendix

A. Proofs for Propositions

A.1 Proof of Proposition 1

Consider the HJB equation as Eq.(12) shows, the marginal utility of staked and non-staked tokens are

$$\begin{aligned} MU_l &= (\mu_t + r_t - c_t + \frac{\partial \Psi}{\partial n_t}) \partial_w J + (x_t + l_t) \sigma_t^2 \partial_{ww} J, \\ MU_x &= \left(\mu_t + (1 - \alpha) \left(\frac{A_t U_t}{x_t} \right)^\alpha + \frac{\partial \Psi}{\partial n_t} \right) \partial_w J + (x_t + l_t) \sigma_t^2 \partial_{ww} J. \end{aligned} \quad (\text{A.1})$$

Note that MU_x and MU_l contains some common terms, including price appreciation μ , loss of numeraire convenience $-\frac{\partial \Psi}{\partial n_t}$ and volatility risk of token price. Agents will always obtain these part of utility once they hold tokens. Therefore, agents make choice between staking and non-staking through comparing the remaining terms, $(r_t - c_t)$ and $(1 - \alpha) \left(\frac{A_t U_t}{x_t} \right)^\alpha$. The marginal return of staked token remains the same, while the marginal utility of non-staked token diminishes to zero. In addition, $x_t \rightarrow 0_+$, the marginal utility of non-staked token must exceed $r_t - c_t$. That is, there exists a unique \tilde{x}_t that satisfies

$$r_t - c_t = (1 - \alpha) \left(\frac{A_t U_t}{\tilde{x}_t} \right)^\alpha. \quad (\text{A.2})$$

The agent will first choose to keep enough tradable tokens for earning transaction convenience. Once her holding of non-staked tokens x_t reaches \tilde{x}_t , she will turn to stake the remaining tokens (if enough tokens are held).

On the other hand, let us consider the aggregate holding of tokens, $q_t = x_t + l_t$. The marginal utility of holding tokens should be the upper envelope of the marginal utility of the two holding ways. Based on the above discussion, we obtain

$$MU_q = \left(\mu_t + \max \left\{ r_t - c_t, (1 - \alpha) \left(\frac{A_t U_t}{q_t} \right)^\alpha \right\} + \frac{\partial \Psi}{\partial n_t} \right) \partial_w J + q_t \sigma_t^2 \partial_{ww} J. \quad (\text{A.3})$$

MU_q decreases strictly with q_t (note that $\partial_w J > 0$ and $\partial_{ww} J < 0$). In addition, when $q_t \rightarrow 0_+$, the \max term tends to positive infinity, while when $q_t \rightarrow w_-$, $n_t \rightarrow 0_+$ and then $-\frac{\partial \Psi}{\partial n_t}$ tends to negative infinity. Therefore, there is a unique q_t^* that satisfies the first order condition, i.e. agent has a unique and positive optimal choice q_t^* .

Rearrange the expression of optimal individual staking ratio θ_t^* ,

$$\theta_t^* = \frac{l_t^*}{q_t^*} = 1 - \frac{x_t^*}{q_t^*} = 1 - \frac{\min\{q_t^*, \tilde{x}_t\}}{q_t^*}, \quad (\text{A.4})$$

where the economic meaning of the minimized term is that when $x_t \leq \tilde{x}_t$, earning transaction convenience is better than earning staking reward, while when $x_t \leq q_t^*$, holding tradable token is better than holding numeraire. Substituting Eq.(A.2), we obtain

$$\theta_t^* = \max \left\{ 0, 1 - \left(\frac{1 - \alpha}{r_t - c_t} \right)^{\frac{1}{\alpha}} \frac{A_t U_t}{q_t^*} \right\}. \quad (\text{A.5})$$

Then Proposition 1 is proved.

Based on the above results, we have the following intuitions.

First, agents' individual staking ratio increase with reward rate r_t . Consider the impact of reward rate r_t . As r_t increases, the marginal utility of staked token increases, while both the convenience of token and numeraire remains the same. Intuitively, agents will increase the proportion of staking. Mathematically, an increase in r_t will cause the *max* term in Eq.(A.3) increases (non-strictly), so that q_t^* increases. Substitute into Eq.(20), we obtain a larger optimal θ_t^* .

Second, agents have heterogeneous optimal staking choices, which are related to their wealth. Note that in the expression of θ_t^* , both U_t and q_t^* are relative to agent's wealth. Therefore, the optimal individual staking ratio may be different among agents due to the difference of wealth level. In numerical analysis, after we make more detailed assumptions on the user type U_t , We will examine the difference between the optimal decisions of agents with different wealth levels. Especially, when U_t increases monotonically in w_t with a diminishing marginal change, the agent who owns more wealth will decide to invest a greater proportion for staked tokens.

A.2 Proof of Proposition 2

We first analyse the resulting overall staking ratio under a given reward rate r_t , $\Theta(r_t)$. Following the proof of Proposition 1, we obtain $x_t^* = \min\{q_t^*, \tilde{x}_t\}$. By Eq.(A.2),

$$\tilde{x}_t = \left(\frac{1 - \alpha}{r_t - c_t} \right)^{\frac{1}{\alpha}} A_t U_t. \quad (\text{A.6})$$

Treat \tilde{x} as a function of individual wealth w and global reward rate r at time t , \tilde{x}_t is differentiable to r and $\frac{\partial \tilde{x}(w, r)}{\partial r} < 0$. On the other hand, when $x_t^* = q_t^*$, i.e. $q_t^* \leq \tilde{x}_t$, the *max* term in Eq.(A.3) equals to $(1 - \alpha) \left(\frac{A_t U_t}{q_t^*} \right)^{\alpha}$. q_t^* solves the first order condition for any given r , thus q_t^* can be treated as a function of r that is shown to be differentiable, $\frac{\partial q_t^*(w, r)}{\partial r} \leq 0$.

Note that $\forall i, w_i \in W$, where $W \equiv [0, \bar{w}]$ is a closed set. We have obtained that $x^*(w, r)$ is differentiable with respect to r at r_j , $\forall w \in W \setminus W_j$, where W_j is defined as, $\forall w \in W_j$, $x^*(w, r_j) = q^*(w, r_j)$, and $\forall r' > r_j$, $x^*(w, r') = \tilde{x}(w, r')$. In fact, x^* may also be differentiable to r at r_j for $w \in W_j$. However, we do not need this stronger condition. Since U is monotonous in w , the measure

of W_j is always zero for any r_j .

Consider the definition of overall staking ratio Θ as Eq.(13) shows, $\Theta(r)$ is differentiable and satisfies

$$\begin{aligned}\Theta'(r) &= \frac{d}{dr} \frac{\int_W l(w, r) m(w) dw}{\int_W q(w, r) m(w) dw} = \frac{(\frac{d}{dr} \int_{W \setminus W_r} l m dw)(\int_W q m dw) - (\int_W l m dw)(\frac{d}{dr} \int_{W \setminus W_r} q m dw)}{(\int_W q m dw)^2} \\ &= \frac{(\int_{W \setminus W_r} \frac{\partial l(w, r)}{\partial r} m dw)(\int_W q m dw) - (\int_W l m dw)(\int_{W \setminus W_r} \frac{\partial q(w, r)}{\partial r} m dw)}{(\int_W q m dw)^2} \\ &= \frac{(\int_{W \setminus W_r} \frac{\partial l}{\partial r} m dw)(\int_W x m dw) - (\int_W l m dw)(\int_{W \setminus W_r} \frac{\partial x}{\partial r} m dw)}{(\int_W q m dw)^2} \geq 0,\end{aligned}\tag{A.7}$$

where the third equal sign holds by Theorem 9.42 in Rudin et al. (1964).

In equilibrium under given positive aggregate reward, ρ , reward rate and staking ratio should satisfy the fixed point problem as Eq.(17), i.e., $r\Theta(r) = \rho > 0$. Note the following properties: (i) $r\Theta(r)$ weakly increases in r , and especially, strictly increases in r for any positive $\Theta(r)$. (ii) $r\Theta(r) = 0$ when r equals zero. (iii) $\lim_{r \rightarrow +\infty} r\Theta(r) = r > \rho$. Therefore, $\forall \rho, 0 < \rho < \infty$, there exists a unique r that satisfies the fixed point problem.

Now considering the case as Proposition 2 describes, $\forall \rho' > \rho > 0$, denote the resulting equilibrium reward rate as r' and r respectively. By the monotonicity of $r\Theta(r)$, we obtain $r' > r$. Then by Eq.(A.7), the resulting equilibrium staking ratio satisfies $\Theta(r') > \Theta(r)$, i.e., $\Theta(\rho') > \Theta(\rho)$.

A.3 Proof of Proposition 3

Since P_t can be separately represented as $P(A_t, Q_t) = \frac{1}{Q_t} V(A_t)$, we can analytically derive the partial differentials of P_t . Substituting the differentials into Eq.(26) and rearranging the equation, we obtain the ordinary differential equation for $V(A_t)$ as Eq.(29) shows.

Consider the lower boundary condition when $A_t \rightarrow 0$. Intuitively, when $A_t = 0$, the platform has no productivity and no agent participates. Therefore, the resulting token price must be zero. Specifically, by Eq.(A.6), $\tilde{x} \rightarrow 0$. That is, agents' individual staking ratio will be close to 1.²⁰ Then the reward rate r_t is close to the reward ratio ρ_t , and $q_t \rightarrow l_t$, for any agents. Substituting into the F.O.C. of the HJB equation, we have

$$0 = \left(\mu_t + \rho_t - c_t + \frac{\partial \Psi_t}{\partial n_t} \right) \frac{\partial_w J}{\partial_{ww} J} + \sigma_t^2 q_t.\tag{A.8}$$

²⁰Here we focus on the case that the reward rate r_t is always larger than staking cost c_t no matter what the overall staking ratio is, i.e., $c_t < \rho_t = \min_{\Theta_t \in [0,1]} r(\rho_t, \Theta_t)$. Otherwise, staking is obviously a “bad” choice for agents.

Integrating w over W and note that $\int_W qmdw = PQ = V$,

$$0 = \mu_t + \rho_t - c_t + I^n + \frac{\sigma_t^2 V_t}{I}, \quad (\text{A.9})$$

where $I = \int_W \frac{\partial_w J}{\partial_{ww} J} m dw$, $I^n = \frac{1}{I} \int_W \frac{\partial \Psi_t}{\partial n_t} \frac{\partial_w J}{\partial_{ww} J} m dw$. Substituting the expression of μ_t and σ_t as Eq.(32),

$$0 = V'(A_t) A_t \mu^A + \frac{1}{2} V''(A_t) (A_t \sigma^A)^2 + (\rho_t - \iota_t - c_t + I^n) V(A_t) + \frac{(V''(A_t) A_t \sigma^A)^2}{I}. \quad (\text{A.10})$$

Let $A_t \rightarrow 0$ and note that $V', V'' < \infty$, $V(A_t)$ should equal to zero to satisfy the above equation. That is, $\lim_{A_t \rightarrow 0} V(A_t) = 0$.

As for the upper boundary, the intuition is that all the wealth will be attracted to the platform when A_t is sufficiently high. We first consider the marginal utility of holding numeraire. Following the previous denotation, $q_t = x_t + l_t$, and $n_t = w_t - q_t$. We obtain

$$MU_n(n_t) = -\mu_t + \min \left\{ c_t - r_t, -(1 - \alpha) \left(\frac{A_t U_t}{w_t - n_t} \right)^\alpha \right\} - \frac{\partial \Psi_t}{\partial n_t} - \frac{(w_t - n_t) \sigma_t^2 \partial_{ww} J}{\partial_w J}. \quad (\text{A.11})$$

We want to show that for any positive $\epsilon < \bar{w}$, there exists $A_t(\epsilon)$ such that $MU_n(\epsilon) < 0$ for any $A_t > A_t(\epsilon)$. There are some useful intuition. First, $\frac{\partial P}{\partial A} \geq 0$. Since both transaction convenience and the aggregate amount of staking reward increases with A_t , a higher A_t will naturally attract more wealth from holding numeraire. Second, $\mu > -\infty$ based on the assumption that $\mu^A \geq 0$. A_t is a process that broadly captures technological advances, regulatory changes, and the variety of activities feasible on the platform, all of which suggest a fast and volatile growth of A_t . Suppose that μ_t tends to negatively infinity, then by Eq.(23), there must be $\frac{\partial^2 P}{\partial A^2} \rightarrow -\infty$, which contradicts the fact that the first order derivative is always greater than zero. This assumption follows Cong et al. (2021d), where the additional reasons for parameter choices also result in a bounded σ_t . Third, $\frac{w \partial_{ww} J}{\partial_w J}$ is bounded since it is a smooth function of $w \in [0, \bar{w}]$. Then $\forall \epsilon \in (0, w_t)$,

$$MU_n(\epsilon) < r^f - \mu - (1 - \alpha) \left(\frac{AU}{w} \right)^\alpha + \psi(\epsilon) - \frac{w \sigma^2 \partial_{ww} J}{\partial_w J}, \quad (\text{A.12})$$

where $\psi(\epsilon) = -\frac{\partial \Psi(\epsilon)}{\partial n} < \infty$. Let

$$A(\epsilon) = \max \left\{ 0, \frac{w}{U} \left(\frac{\psi(\epsilon) + r^f - \underline{\mu} + \bar{J}}{1 - \alpha} \right)^{\frac{1}{\alpha}} \right\}, \quad (\text{A.13})$$

where $\underline{\mu}$ and \bar{J} are the lower bound of μ and upper bound of $-\frac{w \sigma^2 \partial_{ww} J}{\partial_w J}$ respectively, and the max

term insures a non-negative A_t . Substituting $A(\epsilon)$ into Eq.(A.12), we obtain $MU_n(\epsilon) \leq 0$. Note that for the same ϵ , $MU_n(\epsilon)$ decreases with A_t . Therefore, $\forall A_t > A(\epsilon)$, $MU_n(\epsilon) \leq 0$. The result holds for any sufficiently small positive ϵ , which implies that when A_t tends to infinity, the marginal utility of holding numeraire is always negative. Therefore, all the wealth will be allocated to the platform, i.e.

$$\lim_{A_t \rightarrow \infty} V(A_t) = \int_W w_t m(t, w_t) dw_t. \quad (\text{A.14})$$

In the following, we summarize the steps of solving the pricing ODE Eq.(29). First, the equilibrium Θ_t , r_t , and the integral equations for the crowd, are all functions of (A_t, μ_t, σ_t) . Second, when substituting these functions into the market clearing condition, Eq.(25) will only contain P_t , μ_t and σ_t . Third, replacing P_t with V_t by Eq.(28) and apply to Itô's Lemma, we can express μ_t and σ_t by the derivatives of P_t . Then the equation implies a second-order ODE of $V(A_t)$ as Eq.(29) shows. As we mentioned, besides $V(A_t)$, $V'(A_t)$ and $V''(A_t)$, the remaining terms, including Θ_t , I_t , I_t^n and I_t^x are all functions of A_t . Therefore, in the process of numerical solution, we deal with a differential-algebraic system of equations (DAE) in fact. For the boundary condition, we choose a sufficient small ϵ and correspondingly choose $A(\epsilon)$ as Eq.(A.13) shows, so that $V(A(\epsilon)) \in (\int_W w_t m(t, w_t) dw_t - \epsilon, \int_W w_t m(t, w_t) dw_t)$. Let $V(A_t) = \int_W w_t m(t, w_t) dw_t - \epsilon$ and calculate the solution. We then decrease ϵ until the new resulting solution is numerically indistinguishable from the previous solution. Finally, we substitute the solution of $V(A_t)$ and the differentials into Eq.(28) and Eq.(23) to obtain P_t , μ_t and σ_t , and then solve the equilibrium Θ_t , r_t , and the integral equations for the crowd.

A.4 Proof of Proposition 4

In Appendix A.1, we have proved that for any agents with different wealth, there is a unique q^* that satisfies the first order condition as Eq.(A.3) describes. Denote the excess return as λ_t , $\lambda_t = \mu_t + r_t - c_t$. Rearrange Eq.(A.3), we obtain

$$\lambda_t = -\frac{\partial \Psi}{\partial n_t} - \frac{q_t^* \sigma_t^2 \partial_{ww} J}{\partial_w J} + \frac{\min\{0, MU_x^* - MU_l^*\}}{\partial_w J}, \quad (\text{A.15})$$

where MU_l^* and MU_x^* are marginal utility of staked and tradable tokens when agent's controls are optimized. Note that λ_t is a system state that is independent of the controls of a single agent, and the above equation holds for any w_t . Especially, for agents with zero staked tokens ($l_t^* = 0$), $MU_x^* \geq MU_l^*$, we obtain

$$\lambda_t = -\frac{\partial \Psi}{\partial n_t} - \frac{q_t^* \sigma_t^2 \partial_{ww} J}{\partial_w J}, \quad (\text{A.16})$$

which can be interpreted as the trade-off between the transaction convenience of holding tradable token and the convenience of holding numeraire. Substituting into Eq.(A.15), we obtain they staked

token is also compensated with staking rewards as financial returns for the loss of transaction convenience.

B. Extended Discussions

B.1 Joint Dynamic of Staking Ratio and Price Drift

To better understand the dynamic of staking ratio and price drift, we use a schematic as Figure B1 shows to explain the interaction and generation of the equilibrium dynamic. The orange surface plots the equilibrium staking ratio under $(\ln(A_t), \mu_t)$. In fact, the equilibrium staking ratio is determined under $(\ln(A_t), \mu_t, \sigma_t)$. Note that σ_t is determined simultaneously with μ_t by Itô's Lemma, thus we do not need an extra axis for σ_t . The x-axis represents $\mu_t \in [-0.05, 0.05]$, the y-axis represents $\ln(A_t) \in [-15, 15]$, and the z-axis represents $\Theta_t \in [0, 1]$. Even though we have not solve the equilibrium yet, we can enumerate all possible cases of $(\ln(A_t), \mu_t)$ and solving the corresponding staking ratio as the orange surface shows.²¹ Given μ_t , as A_t increases, the staking ratio decreases. On the other hand, under a fixed A_t , staking ratio increases with μ_t , especially when A_t is small. These conclusions are reasonable from agents' perspective. First, when A_t increases, transaction convenience is gradually higher, then agents will allocate more tradable tokens. Second, agents also compare the tokens with numeraire. Higher price drift leads to higher excess returns for holding tokens (both tradable and staked tokens). Especially, when A_t is low, agents will be more likely to stake than to hold tradable tokens.

Note that the points on the orange surface, $(\ln(A_t), \mu_t, \Theta(\ln(A_t), \mu_t))$, satisfies agents' optimization and fixed point problem of r_t , but not all the points satisfies the pricing ODE. However, it is feasible to check whether a particular point satisfies the ODE based on boundary conditions.²² Then we derive one particular curve that satisfies the pricing ODE as the orange line shows. From another perspective, only the points on the grey line, $(\ln(A_t), \mu(A_t, V(A_t), V'(A_t), V''(A_t)))$, satisfies the pricing ODE. Then we draw a cutting plane that perpendicular to xOy as the grey surface shows. The intersection line of two surfaces is the system solution.²³ The blue line is the projection on plane xOz of the solution, i.e., the joint dynamic of staking ratio and price drift as Figure 3 shows.

²¹In fact, we need three dimensions to actually enumerate all the cases. To simplify the plot, we use the "correct" corresponding σ of each grid $(\ln(A_t), \mu_t)$.

²²For example, given $\ln(A_t) = -15$, $\mu_t = -0.05$ (and $\sigma_t = 2.00$), we obtain the resulting staking ratio (around 0.4) and other integrals for the crowd including $V(A_t)$. If we have determined the value of $V(A_t)$ near $\ln(A_t) = -15$, then we can derive the resulting μ'_t and σ'_t . Such point satisfies the ODE if and only if $\mu'_t = \mu_t = -0.05$ and $\sigma'_t = \sigma_t = 2.00$. This is actually the mechanism in the numerically solving process.

²³This is just a more comprehensible perspective. In fact the determination of the gray curve depends on the solution of the agent's optimization problem.

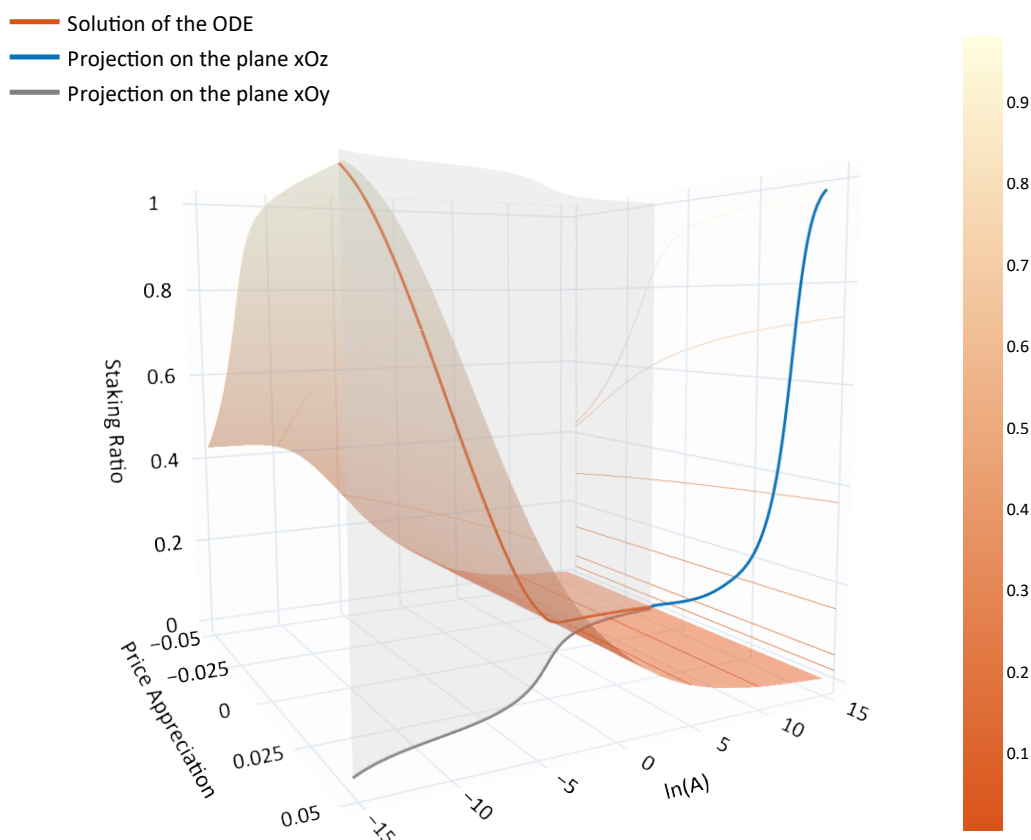


Figure B1: Platform productivity, staking ratio, and expected price drift.

The figure visualizes the relationship among expected price appreciation μ_t , platform productivity A_t , and staking ratio Θ_t . The x-axis represents $\mu_t \in [-0.05, 0.05]$, the y-axis represents $\ln(A_t) \in [-15, 15]$, and the z-axis represents $\Theta_t \in [0, 1]$. The orange surface plots the equilibrium staking ratio under $(\ln(A_t), \mu_t)$. (In fact, the equilibrium staking ratio is determined under $(\ln(A_t), \mu_t, \sigma_t)$. Note that σ_t is determined simultaneously with μ_t by Itô's Lemma, thus we do not need an extra axis for σ_t .) The orange curve on the surface is the solution that satisfies the pricing ODE. The blue and grey lines are respectively the projection of the orange curve on plane xOz and xOy. The grey surface is a cutting plane perpendicular to xOy and crosses the orange surface with the orange curve.

C. Mean Field Game (of controls) System

Mean field game (MFG) is a powerful tool to analyse strategic interactions in large populations when each individual agent has only a small impact on the behavior of other players, which has been introduced in the pioneering works of Lasry and Lions (2007). MFG supposes that the rational agents are indistinguishable and individually have a negligible influence on the game, and that each individual strategy is influenced by some averages of quantities depending on the states of the other agents. As an extended form of MFG, the mean field game of controls (MFGC) system exhibits interaction among the agents through not only their state but also their actions (controls). In our research, staking reward rate is influenced by aggregate staking ratio, which is hence connected to wealth distribution and agents controls (individual staking choices). Therefore, the MFGC approach is used in our model.

The MFGC in the present work is described by a system of nonlocal partial differential equations. It contains a Hamilton-Jacobi-Bellman (HJB) equation that leads to the optimal control of agents, a Fokker-Planck-Kolmogorov (FP) equation that describes the transport-diffusion of the distribution of states, and respectively a fixed point problem which gives a connection among states, controls and mean field distribution. In our model, Eq.(12), (16) and (17) make up the whole system.

On the mathematical analysis of MFG, Lasry and Lions (2007) gives a proof of existence and uniqueness of solutions under sufficient assumptions such as monotonicity. It is possible to extend these arguments to MFGC, see Cardaliaguet and Lehalle (2018). In solving the model, we followed these sufficient assumptions and used the numerical solution method proposed by Achdou and Kobeissi (2020).

D. Parameter Choices

In the numerical analysis, we set the initial wealth distribution to be to follow the Pareto distribution with parameters $w_{min} = 10$ and $k = 3$. Such distribution fits the trend that a large portion of wealth is held by a small fraction of the population. For numerical test, we set the maximum wealth to be $w_{max} = 100$, which is sufficient for discussion on heterogeneous optimal choice, and the corresponding value of cumulative function has already reached $1 - 10^{-3}$. Since there are unit measure of agents, the initial total wealth equals $\mathbb{E}(w) = 15$. We set the initial amount of tokens Q_0 to be 15, which makes the token price to be 1 approximately when all the wealth flows into the platform. It is just to get a simple number without affecting any analysis process. For example, token price is halved when the total amount of tokens is doubled, while the equilibrium dynamics is invariant. We set the inflation rate ι fixed at 3%. The value is taken with reference to the actual issuance of tokens. On the one hand, the fixed value matches the setting of a large

part of the tokens, that are designed to have a constant inflation rate, such as eos. On the other hand, the constant set makes the model easier to solve so that we can focus on the main interests. The rewards from transaction fee is defined to be a random variable. Numerically, we exogenously set different values of τ ranges from 0 to 0.05. The resulting solutions satisfy Proposition 2. At the same inflation rate, larger τ leads to larger amount of reward, and thus generates a larger equilibrium staking ratio and reward rate.

We set the annual risk-free rate of numeraire, r^f , constantly equals to be zero. Then we choose $\mu^A = 2\%$. As Cong et al. (2021d) discusses, A_t broadly captures regulatory changes, and the variety of activities feasible on the platform, which suggest a volatile growth of A_t . We set $\sigma^A = 200\%$.

For parameters of agents, we set the instant utility to be $\mathcal{U}(y) = \frac{y^{1-\gamma}-1}{1-\gamma}$ with $\gamma = 0.9$, so that agents exhibits constant risk aversion $\gamma = 0.9$ the elasticity of intertemporal substitution $1/\gamma$. The user type $U = U(w)$ reflects agents' transaction demand. We set $U(w) = \kappa w^\delta$ with $\kappa = 0.01$ and $\delta = 0.1$. This setting is to satisfy the natural assumption that $\frac{\partial U}{\partial w} > 0$, $\partial^2 U / \partial w^2 < 0$. The specific values taken have little effect on the main conclusions. For example, a larger κ makes the transaction convenience of all agents increases with constant A , but the direction and nature of the qualitative propositions does not change. We set $\alpha = 0.3$, which adjusts the sensitivity of the agent to the platform productivity. In Cong et al. (2021d), α is also set to be 0.3 to match the data. For the convenience of numeraire, we follow Valchev (2020) to model the consumption cost as $\Psi(y, n) = \bar{\psi} y^\beta n^{1-\beta}$, where $\beta > 1$ features that costs are increasing in consumption, and decreasing in the level of numeraire holdings. We follow Valchev's value choice, $\beta = 18$ and $\bar{\psi} = 4.2e - 18$, that are aimed to match the interest rate semi-elasticity of money demand. Our study does not address these concepts. We simply follow the value choices to confirm the reasonableness in reality. In fact, in our study, the main impact of this term lies in the convenience gain of holding numeraire. With the guarantee that $\partial \Psi / \partial n < 0$, the specific choice of the relevant parameter does not affect the main properties.

E. Staking Mechanism of Tokens in Our Sample

In the following, we describe representative staking mechanisms of some tokens in our sample. Most information are accessed from *Stakingrewards.com*. There is also information from official websites of corresponding tokens. Many tokens have similar mechanisms, thus we do not repeat the description.

- The individual AION rewards depends on the Block Reward, Block Time, Daily Network Rewards and Total Staked. Every block one validator is randomly selected to create a block, whereas 1 staked or delegated token counts as one "lottery ticket". The selected validator has the right to create a new block and broadcast them to the network. The Validator

then receives the 50% of the block reward and the fees of all transactions (network rewards) successfully included in this block, whereas the PoW Miner receives the other 50%.

- Rewards in the form of algos are granted to Algorand users for a variety of purposes. Initially, for every block that is minted, every user in Algorand receives an amount of rewards proportional to their stake in order to establish a large user base and distribute stake among many parties. As the network evolves, the Algorand Foundation will introduce additional rewards in order to promote behavior that strengthens the network, such as running nodes and proposing blocks.
- The individual BitBay rewards depends on the Block Reward, Block Time, Daily Network Rewards and Total Staked. Every block one staker is randomly selected whereas 1 staked coin counts as one “lottery ticket”. The selected staker has the right to create a new block and broadcast it to the network. He then receives the block reward and the fees of all transactions successfully included in this block.
- Dash blockchain consensus is achieved via Proof of Work + Masternodes. Investors can leverage their crypto via operating masternodes. Miners are rewarded for securing the blockchain and masternodes are rewarded for validating, storing and serving the blockchain to users.
- Eos has a fixed 5% annual inflation. 4% goes to a savings fund, which might distribute the funds to the community later on. 1% goes to Block producers and Standby Block Producers. Out of the 1% that are given to block producers, only 0.25% will go to the actual 21 producers of the blocks. The other 0.75% will be shared amongst all block producers and standby block producers based on how many votes they receive and with a minimum of 100 EOS/day.
- The individual reward of staking fantom depends on the Total Staked ratio. Transactions are packaged into event blocks. In order for event blocks to achieve finality, event blocks are passed between validators nodes that represent at least 2/3rds of the total validating power of the network. A validator’s total validating power is primarily determine by the number of tokens staked and delegated to it. A validator earns rewards each epoch for each event block signed according to it’s validating power. By delegating investors can increase the share of your validator proportionally to the balance of your account. He will receive rewards accordingly and share them with investors after taking the commission.
- The effective yield for staking IDEX depends on the actual Trading Volume on IDEX Market. The higher the trading volume on IDEX, the higher are the actual rewards. The second metric to watch is the total amount of AURA currently staking. Less tokens on stake result in higher rewards.

- Every livepeer (LPT) token holder has the right to delegate their tokens to an Orchestrator node for the right to receive both inflationary rewards in LPT and fees denominated in ETH from work completed by that node.
- The individual LTO rewards depends on the Network Rewards (Transaction Fees spent on the Network) and the Total Staked. Every block one staking node operator is randomly selected to create a new block, whereas 1 staked token counts as one “lottery ticket”. The staker receives the fees of all transactions successfully included in this block. Staking Node Operators share the rewards with their delegators after deducting a commission.
- NEM blockchain consensus is achieved via Proof of Importance. Investors can leverage their crypto via harvesting. To harvest NEM coins it is recommended to run the official NEM Core wallet with an entire copy of the blockchain on your Computer or a Virtual Private Server (VPS). The individual NEM harvesting rewards depends on the Daily Network Rewards and Total Staked. Every block one staker is randomly selected whereas 1 staked coin counts as one “lottery ticket”. The selected staker has the right to create a new block and broadcast it to the network. He then receives the fees of all transactions successfully included in this block.
- Everyone who holds NEO will automatically be rewarded by GAS. GAS is produced with each new block. In the first year, each new block generates 8 GAS, and then decreases every year until each block generates 1 GAS. This generation mechanism will be maintained until the total amount of GAS reaches 100 million and no new GAS will be generated.
- Nuls blockchain consensus is achieved via Proof of Stake + Masternodes. Investors can leverage their crypto via staking. The amount earned is variable based on the current blockchain metrics like the amount of stakers (Total Staked ratio). You can stake NULS into a project’s nodes and earn their token as a reward, while the project earns NULS as a reward. Some projects offer to stake with just 5 NULS as the minimum.
- Delegators in Polkadot are called Nominators. Anyone can nominate up to 16 validators, who share rewards if they are elected into the active validators set. The process is a single-click operation inside the wallet. Simply choose 1-16 validators (staking providers) who you trust and nominate them. The current reward rate for validators is determined by the current Total Staked ratio. The less DOT is being staked, the higher are the rewards.
- Qtum blockchain consensus is achieved via Proof of Stake 3.0. The individual reward depends on the Block Reward, Block Time, Daily Network Rewards and Total Staked. Every block one staker is randomly selected whereas 1 staked coin counts as one “lottery ticket”. The

selected staker has the right to create a new block and broadcast it to the network. He then receives the block reward and the fees of all transactions successfully included in this block.

- Synthetix Network Token blockchain consensus is achieved via the Ethereum Blockchain. Investors can leverage their crypto via staking. SNX holders can lock their SNX as collateral to stake the system. Synths are minted into the market against the value of the locked SNX, where they can be used for a variety of purposes including trading and remittance. All Synth trades on Synthetix Exchange generate fees that are distributed to SNX holders, rewarding them for staking the system.
- Tezos blockchain consensus is achieved via Liquid Proof of Stake. Investors can leverage their crypto via baking or delegating. There are a number of tokens that use a similar mechanism, including iotex, irisnet, etc.
- The individual tron rewards depends on the Block Rewards, Endorsement Rewards, Block Time, Daily Network Rewards and Total Staked. Every block one staker is randomly selected to bake a block and 32 stakers are selected to endorse a block, whereas 1 staked coin counts as one “lottery ticket”. The selected stakers have the right to create or endorse new block and broadcast them network. The Baker then receives the block reward and the fees of all transactions successfully included in this block. The Endorsers receive the endorsement rewards.
- Wanchain blockchain consensus is achieved via Galaxy Proof-of-Stake. The individual WAN rewards depends on the Foundation Rewards, Daily Network Rewards and Total Staked. At the beginning of each protocol cycle (epoch), two groups, the RNP (Random Number Proposer) group and the EL (Epoch Leader) group, are selected from all validators. 1 staked or delegated token counts as one “lottery ticket” to be selected. The two groups equally share the Foundation Rewards and Transaction Fees (Network Rewards). The Foundation Rewards consists of 10% of the outstanding Wanchain Token Supply and are decreasing by 13.6% each year, whereas the Network Rewards are expected to rise alongside wider network usage.